1) Suppose we have solved a system of linear equations and determined that the solution set can be described as \( x_1 = 5 + 4x_3, \ x_2 = -2 - 7x_3, \) and \( x_3 \) is free. Use vectors to describe this solution set as a line in \( \mathbb{R}^3 \).

2) Give a geometric description (i.e. sketch the set of solutions as I did in class) of the solution set to the homogeneous linear system:

\[
\begin{align*}
2x_1 + 2x_2 + 4x_3 &= 0 \\
-4x_1 - 4x_2 - 8x_3 &= 0 \\
-3x_2 - 3x_3 &= 0.
\end{align*}
\]

3) Give a geometric description (i.e. sketch the set of solutions as I did in class) of the solution set to the nonhomogeneous linear system:

\[
\begin{align*}
2x_1 + 2x_2 + 4x_3 &= 8 \\
-4x_1 - 4x_2 - 8x_3 &= -16 \\
-3x_2 - 3x_3 &= 12.
\end{align*}
\]

Compare your geometric description to that in problem 2.

4) Determine whether the following statements are true or false. Justify your answer.

a) A homogeneous linear system always has a solution.

b) A homogeneous linear system cannot have more than one solution.

c) If \( x = 0 \) is a solution to the linear system \( Ax = b \), then \( b \) must be the zero vector.

5) Solve the linear systems with the corresponding augmented matrices given below by first putting the augmented matrices in reduced echelon form. Express the solution set as a collection of vectors.

a) 
\[
C = \begin{pmatrix}
1 & -3 & -5 & | & 0 \\
0 & 1 & 1 & | & 3
\end{pmatrix}
\]

b) 
\[
D = \begin{pmatrix}
1 & -2 & 1 & | & 0 \\
0 & 2 & -8 & | & 8 \\
-4 & 5 & 9 & | & -9
\end{pmatrix}
\]

6) Set

\[
A = \begin{pmatrix}
2 & 0 & -1 \\
4 & -5 & 2
\end{pmatrix}, \quad B = \begin{pmatrix}
7 & -5 & 1 \\
1 & -4 & -3
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
2 & 0 & -1 \\
4 & -5 & 2
\end{pmatrix}, \quad B = \begin{pmatrix}
7 & -5 & 1 \\
1 & -4 & -3
\end{pmatrix}
\]
\[ C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 5 \\ -1 & 4 \end{pmatrix}. \]

Compute the following, if they are defined. If they are undefined, explain why:

(a) \(-2A\)
(b) \(B - 2A\)
(c) \(AC\)
(d) \(CD\).

Problems 7-8: Compute the product \(AB\) for the matrices given below using two different methods: 1) by computing \(Ab_1\) and \(Ab_2\) separately, and 2) by taking the dot product of columns with rows to determine the entries of \(AB\).

7)
\[
A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \\ 5 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix}.
\]

8)
\[
A = \begin{pmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}.
\]

9) If \(A = \begin{pmatrix} 1 & -3 \\ -3 & 5 \end{pmatrix}\), and \(AB = \begin{pmatrix} -3 & -11 \\ 1 & 17 \end{pmatrix}\), determine the first and second columns of \(B\).

10) Suppose \(CA = I_{n \times n}\), where \(I_{n \times n}\) is the \(n \times n\) identity matrix. Show that the matrix equation \(Ax = 0\) has only the trivial solution \(x = 0\).