1) Determine whether the following matrices are invertible:

\[ A = \begin{pmatrix} 3 & -9 \\ 2 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & -9 \\ -4 & 6 \end{pmatrix}. \]

2) Find the inverse of the matrix \( A = \begin{pmatrix} 7 & 3 \\ -6 & -3 \end{pmatrix}. \)

3) Use your answer to Problem 2 to solve the linear system

\[
7x_1 + 3x_2 = -9 \\
-6x_1 - 3x_2 = 4.
\]

4) State whether the following are true or false. Explain your answer.

a) If \( A \) and \( B \) are both \( n \times n \) invertible matrices, then \( A^{-1}B^{-1} \) is the inverse of \( AB \).

b) If \( A \) is an \( n \times n \) invertible matrix, and \( B \) and \( C \) are both \( n \times p \) matrices satisfying \( AB = AC \), then \( B = C \).

5) State whether the following are true or false. Explain your answer.

a) Every elementary reduction matrix is invertible.

b) If \( A \) is an invertible \( n \times n \) matrix, then the elementary row operations that reduce \( A \) to \( I_{n \times n} \) also reduce \( A^{-1} \) to \( I_{n \times n} \).

6) Use matrix algebra to show that if \( A \) is an invertible \( n \times n \) matrix and \( D \) is a matrix which satisfies \( AD = I_{n \times n} \), then \( D = A^{-1} \). (Note what this statement is saying: if you already know that \( A \) is invertible, and you find a matrix \( D \) satisfying \( AD = I_{n \times n} \), then you can conclude that \( D = A^{-1} \) without checking to see if \( DA = I_{n \times n} \) as well.)

7) Find the inverse of the following matrix, if it exists:

\[ C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}. \]

8) Find the inverse of the following matrix, if it exists:

\[ D = \begin{pmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{pmatrix}. \]
9) Let
\[ A = \begin{pmatrix} -1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{pmatrix}. \]
Find the third column of \( A^{-1} \) without computing the other columns.

10) Suppose \( A \) is an \( n \times n \) matrix and that the equation \( A\mathbf{x} = \mathbf{0} \) has only the trivial solution \( \mathbf{x} = \mathbf{0} \). Explain why \( A \), when put in reduced echelon form, is the identity matrix \( I_{n \times n} \).

11) Suppose \( A \) and \( B \) are \( n \times n \) matrices, \( B \) is invertible, and \( AB \) is invertible. Show that \( A \) is invertible. (Hint: Let \( C = AB \), and solve this equation for \( A \).)