MTH 341 - Homework 8 (due Friday, December 4)

1) Find the characteristic equation of the matrix
\[
\begin{pmatrix}
3 & 1 & 1 \\
0 & 5 & 0 \\
-2 & 0 & 7
\end{pmatrix}
\].

2) Find the characteristic equation of the matrix
\[
\begin{pmatrix}
4 & 0 & -1 \\
-1 & 0 & 4 \\
0 & 2 & 3
\end{pmatrix}
\].

3) For the matrix below, list the eigenvalues and state the multiplicity of each:
\[
\begin{pmatrix}
3 & 0 & 0 & 0 \\
6 & 2 & 0 & 0 \\
0 & 3 & 6 & 0 \\
2 & 3 & 3 & -5
\end{pmatrix}
\].

4) True/False. Justify your answer.

(a) An elementary row operation on \( A \) does not change its eigenvalues.

(b) An elementary row operation on \( A \) does not change its determinant.

5) Show that if \( A = QR \) with \( Q \) invertible, then \( A \) is similar to \( RQ \).

6) Show that if \( A \) and \( B \) are similar, then \( \det A = \det B \).

7) Consider the linear transformation defined by \( T(x) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \). Describe geometrically what \( T \) does to each vector \( x \) in \( \mathbb{R}^2 \).

8) Consider the linear transformation defined by \( T(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \). Describe geometrically what \( T \) does to each vector \( x \) in \( \mathbb{R}^2 \).

9) Let \( T \) be a linear transformation that maps \( u = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \) to \( \begin{pmatrix} 4 \\ 1 \end{pmatrix} \) and maps \( v = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \) to \( \begin{pmatrix} -1 \\ 3 \end{pmatrix} \). Determine \( T(2u) \), \( T(3v) \), and \( T(2u + 3v) \).

10) Assume \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) is a linear transformation satisfying \( T(e_1) = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \), \( T(e_2) = \begin{pmatrix} -2 \\ 9 \end{pmatrix} \), and \( T(e_3) = \begin{pmatrix} 3 \\ -8 \end{pmatrix} \), where \( e_1 \), \( e_2 \), and \( e_3 \) are the columns of
$I_{3\times 3}$. Find the standard matrix of $T$.

11) Determine if the linear transformation $T$ given in exercise 10 is

   (a) one-to-one.

   (b) onto.

12) (a) Find the standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates points in the clockwise direction $-3\pi/2$ radians (rotation is about the origin).

   (b) Find the standard matrix for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which first rotates points in the clockwise direction $-3\pi/4$ radians, and then reflects points through the horizontal $x_1$-axis.