This report covers the time period from August 2004 through December 2005.

The purpose of this grant is to adapt Ken Bogart’s successful project, “Teaching Introductory Combinatorics by Guided Group Discovery”, for required discrete mathematics courses for math majors. Ken was a consultant on this project, and was intimately involved from its inception in June 2003 until his death in March 2005. In the past half-year, one of my activities has been to identify ways the project has changed since Ken’s death and to try to compensate for important deficiencies within budgetary constraints. John Lee (last term’s instructor of our course) has become very involved in the project, sharing my enthusiasm for both the course and its continued adaptation. John is an invaluable asset to both the course and the project and I’ve included more specifics on that in later parts of this report. I’ve decided the project would benefit from an objective assessment of how the adaptation departs from Ken’s original as well as an informed opinion on which changes have resulted in problem sequences which are no longer in the spirit of guided discovery. I’ve corresponded with some members of Ken’s advisory board because I think this can best be done by someone who was involved in his original project. Although I can’t report any definite conclusion, I have reason to hope that this deficiency will be remedied as much as the grant budget allows.

An overview of the activities from August 2004 to December 2005:

2. Fall 2004: Class taught by Mary Flahive and evaluated by Barbara Edwards.
3. Fall 2004: Ken and I prepared and submitted a minicourse proposal for the Winter 2006 meetings which was not accepted.
5. Winter to Summer 2005: Adaptation of the notes for the Fall 2005 class.
6. Spring 2005: Construction of the website for the project.
7. Summer 2005: Submission of a manuscript for a chapter in Resources for Teaching Discrete Mathematics, a volume in the MAA Notes series. (I met the editor, Brian Hopkins, during the Jan 2005 poster session when he requested the chapter.) I’m now in the process of updating the chapter.
8. Fall 2005: Class taught by John Lee and evaluated by Barbara Edwards. (The evaluation has not yet been completed.)
9. In Fall 2005 and Winter 2006 I’m influenced by this project while I’m teaching the
discrete math sequence for CS majors. (85 students in Fall 2005 and 55 students
in Winter 2006, with non-empty intersection)

Before giving more specifics on the adaptation of Ken’s notes and method in the
2004 and 2005 classes, I think it would be helpful to review my initial interest in this.
After learning about Ken’s notes from a SIAM discrete mathematics newsletter in
Spring 2003 and attending the Bogart-Collins workshop in August 2003, I was eager
to see how the notes would be received by OSU’s Fall 2003 class, an environment
which was significantly different from the ones in which it had been tested. Among
the differences Ken and I identified were: larger-sized class (usually 24-30 students);
more heterogeneous mathematical background and motivation; the more extensive
syllabus of discrete mathematics rather than enumerative combinatorics. It might
be worthwhile to investigate whether or not our initial assumptions were warranted
and to identify other differences, but this is our working list of the most important
differences.

As the Fall 2003 quarter evolved, I was impressed by the amount of personal
responsibility shown by many of the students and by the enthusiasm generated by
many of the problem sequences. Some of this can be attributed to the type of material,
but I’m convinced that most of the energy comes from the Bogart method, the choice
of topics, and the problem sequencing. It is important to maintain these elements of
discovery and community while adding some more elementary material and expanding
the written guidance (which is helpful in a larger class with six or seven groups and
one instructor.)

The instructor in Fall 2005, John Lee, frequently teaches in the advanced calculus
sequence which math majors usually begin the same term as they take this course.
John is especially impressed by the accessibility of the topics in this course and in
that regard thinks the problem sequences have been designed well even for groups of
students with differing amounts of mathematical training. Given this accessibility, he
also found that the notes fostered and developed important mathematical instincts in
different ways from other math major courses he’s taught. Among these are: checking
small cases first; formulating and testing conjectures; the importance of testing for
counterexamples in combination with trying to construct a proof. John has become
heavily involved in the course and adaptation. In addition, each of us is incorporating
features of the Bogart method into other courses, and that experimenting in turn
informs the continuing adaptation. John will teach the next offering of the course in
Fall 2006.

The remainder of this report will give specifics of the two adaptations of the notes
(Fall 2004 and Fall 2005) and also give highlights of the two course implementations
which have occurred during the time period of this report.

In the first adaptation (Fall 2004) I made the modifications which I had identified
as necessary during my use of the original notes with the Fall 2003 class. In particu-
lar, in order to compensate for the higher student/faculty ratio and heterogeneity of student background there was a general supplementing of material, including added exposition and motivational material for guidance. Since some students were uncomfortable with the amount of material in Ken’s first chapter, that material was spread over three chapters: Beginning combinatorics (an outline of basic principles); The Product Principle, Revisited (included the more general principle as well as the Quotient Principle and equivalence relations); The Bijection Principle, Revisited (including Catalan numbers and counting labelled trees). This new presentation encouraged the less mathematically-sophisticated students to review what they’ve learned more frequently, and also facilitated that summarizing process by packaging the material in more obvious units. Our course could not assume the elementary material contained in the appendices (principally, functions, equivalence relations, and induction) as a prerequisite for the course. On the other hand, there was sufficient overlap between the appendices and the main notes that simply covering the appendices first didn’t work. Finally, because our course has the more general syllabus of discrete mathematics rather than enumerative combinatorics, some advanced counting was removed to supplementary sections. Ken was involved with all these changes.

The Fall 2004 adaptation placed induction in about the middle of the quarter, much later than induction was used in their concurrent course in advanced calculus. I discovered that many of the students were associating induction with formulaic exercises (such as finding a closed form for the sum of the first $n$ positive integers) rather than as a method for proving a sequence of statements indexed by the positive integers. They needed experience in identifying the underlying inductive process in order to be able to understand how to set up and prove the more varied problems in these notes. Both Ken and I (and now John Lee) were intrigued by this from a pedagogical point of view as well as for the more practical problem of how to address this in the notes. In the Fall 2005 adaptation induction has been moved much earlier (it forms the second chapter). More problems have been added, including a new problem sequence which specifically addresses their understanding of inductive processes. John and I are also working on an exposition of our observations with the view to publication.

Another significant difference between the original notes and the most recent adaptation is the emphasis on considering counting from the perspective of equivalence relations rather than relying on the Quotient Principle. In addition to having wider application to the case of equivalence classes of unequal size, using equivalence relations is an important skill to be obtained from a beginning course in discrete mathematics. The current edition has a full chapter on equivalence relations, absorbing much of the material from the original appendix as well as problem sequences using the Quotient Principle. Ken supported this as an important change for a course in discrete mathematics.

Ken’s advisory board and evaluator advocated for the inclusion of more summarizing material. With the expectations that come with being a required course, we find this to be especially necessary in our course. Also, the students don’t have sufficient
mathematical maturity to ferret out definitions and theorems from the exercises. In addition to more summarizing material in the notes, I’ve written separate summarizing handouts for each chapter which can be modified by each instructor. John found these helpful, and especially liked the problem sequence which now ends the first chapter, a sequence which is designed to help the students do their own summarizing.

As well as adapting the notes, I plan an instructor package which would include a summary of the elements of our modifications of the Bogart method as well as anecdotal advice collected during this project from the instructors and students. Working on the chapter for the volume in the MAA Notes is a start on this, and I expect to devote more time to this package during this next year.

The remainder of this report will give highlights of the two course implementations which have occurred. First of all, I’d like to summarize features of the overall setup which were common to both classes and which will continue next year. Beginning in Fall 2004, the course has been taught in two 100-minute blocks with group work occasionally peppered with whole-class activities. Students are expected to work together but to write up their final solutions independently. Because the students are likely to initially group themselves with people they already know, for the first few weeks of each term group membership was changed by the instructor once a week in order to maximize the opportunity for the students to work with students they might not know well. After the middle of the term, the groups were stable except for the changes caused by occasional absences, and the canonical group size was 4 or 5 students. Since John and I agree that quick and frequent feedback is essential, we each collected written work once a week and provided a 2-day turnaround on graded work. Ken’s method allows for unlimited re-submission of problems, but we restricted both their number and the re-submittal window. We also changed Ken’s 0-5-9-10 “triage” grading-scale slightly by adding a possible grade of 7, partly as a compensation for fewer re-submittals.

We both found offering the course in two long blocks an advantage since it did a good job of modeling how we expected students to work outside the classroom, taking the time to be suitably immersed in problems. However, especially with 25 or more students it can be difficult for everyone to remain on task for that length of time and each of us used whole-class discussion time to help break up the time. Mary used this type of time sparingly during the term, and usually at the 85-minute mark when students seemed especially tired or frustrated, whereas John regularly used it at the 60-minute mark. Next year John plans to try to incorporate more student presentations. Neither of us used this time for anything like a lecture, but rather it usually modelled a successful group with no all-knowing participant. Having said that, we each came to every class with an idea of what we wanted the students to take away from the class time and sometimes the whole-group time was spent guiding the students in a certain direction.

Here are some observations from correspondence with Ken after the first time I taught the course: Many students proceeded at a snail’s pace. Especially in a larger
class, it seems helpful to have most students begin and end the material at roughly the same time. What can I do to motivate the good students to stretch? These questions exemplify problems in obtaining a balance between giving students the independence to work successfully in groups with sufficient instructor guidance to move things along.

Most of the material in the elementary appendices of the original notes was necessary for our students, but flipping back and forth from those sections to the main book interfered with the students’ appreciation of what they were doing and how things fit together. In the adaptations, I’ve incorporated a sufficient amount of the basic material on relations and mathematical induction into the fabric of the early chapters so that an average student doesn’t need to do more than scan the three current review appendices.

What about motivating students to do more? In the first two adaptations some advanced counting (for instance, Ramsey numbers) was moved to supplementary sections. Ken agreed with this move, writing that it might remove “some roadblocks for standard students and to provide intellectual challenge for the ‘honors-level’ students.” In the Fall 2004 I used a special course-grade protocol in an attempt to entice an advanced group who would work more problems. I encouraged three students to consider doing this, and two accepted the challenge. ¹ They worked together in a group of three, with one other student who wasn’t as strong but who enjoyed being a member of the group. As the term evolved, each week these students and I would come to an agreement on what they would hand, and the stronger two students substituted a final problem set for the final. In John’s class, every group proceeded at the same rate, and we’ve decided the next adaptation will move some of the supplementary material as optional sections in the main body of the book. It’s hoped that this move will encourage more students to look at more material.

For the duration of this project, our classes meet on Monday and Wednesday afternoons and the usual format for the classes has been to assign a range of problem numbers—a minimal assignment to be completed by the beginning of class on the following Monday. Although for the beginning material it was reasonable to expect the students to work about 20 problems a week, later problems were harder and their solution required making connections with earlier information. On average, 12-15 problems per week was the usual expectation and this required a healthy amount of time outside the classroom. Occasionally each of us modified the list at the end of the week, but in general we saw the wisdom in setting expectations which the students could strive to meet.

As said earlier, students were encouraged to work together outside class, but the final written draft of their problem solutions was expected to be done independently.

¹At the end of this class one of these students wrote: “Thank you for allowing my small group and me to work separately from the rest of the class. I found it both interesting and educational. My life has been spent in the Oregon school system, and I have been a guinea pig through lots of failed experiments (e.g. the CIM and CAM). The new style of learning in the class notes is the first experimental teaching style that I have really liked. Maybe it could be used beyond Math 399.”
What problems were graded? In the first two-thirds of the term, my students were expected to turn in all assigned problems, and I graded about 5 problems per week using our modified triage scale. As the term evolved I expected the students to better understand the amount of work needed to write a proper mathematical solution, and because of this I changed this procedure by selecting a subset of the assigned problems for them to turn in. On the other hand, at the end of each week John would specify which problems he would grade on the following Tuesday, and he diligently assigned two grades for each problem he graded, with one grade specifically assessing the mathematical exposition.

Both adaptations are available on my website, and this term I expect to add a PDF of the 2005 adaptation which will be annotated in color to indicate the changes anticipated to be made in the 2006 edition. Information for instructors will be gathered and added to the site during this coming year.