1) The random variable $N$ has PMF

$$P_N(n) = \begin{cases} c(1/2)^n, & n = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (1)

(a) What is the value of the constant $c$?
(b) What is $P[N \leq 1]$?

2) The random variable $V$ has PMF

$$P_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

(a) What is the value of the constant $c$?
(b) Find $P[V \leq u^2 | u = 1, 2, 3, ...]$.
(c) Find the probability that $V$ is an even number.
(d) Find $P[V > 2]$.

3) When a conventional paging system transmits a message, the probability that the message will be received by the pager it is sent to is $p$. To be confident that a message is received at least once, a system transmits the message $n$ times.
(a) Assuming all transmissions are independent, what is the PMF of $K$, the number of times the pager receives the same message?
(b) Assume $p=0.8$. What is the minimum value of $n$ that produces a probability of 0.95 of receiving the message at least once?

4) Anytime a child throws a Frisbee, the child’s dog catches the Frisbee with probability $p$, independent of whether the Frisbee is caught on any previous throw. When the dog catches the Frisbee, it runs away with Frisbee, never to be seen again. The child continues to throw the Frisbee until the dog catches it. Let $X$ denote the number of times the Frisbee is thrown.
(a) What is the PMF $P_X(x)$?
(b) If $p=0.2$, what is the probability that the child will throw the Frisbee more than four times?

5) When a two-way paging system transmits a message, the probability that the message will be received by the pager it is sent to is $p$. When the pager receives the message, it transmits an acknowledgment signal (ACK) to the paging system. If the paging system does not receive the ACK, it sends the message again.
(a) What is the PMF of $N$, the number of times the system sends the same message?
(b) The paging company wants to limit the number of times it has to send the same message. It has a goal of $P[N \leq 3] \geq 0.95$. What is the minimum value of $p$ necessary to achieve the goal?