ECE353: Probability and Random Processes

Lecture 1 - Introduction

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What is probability?

• We talk about probability every day in life:
  – e.g., “it is probably gonna rain tomorrow”, “my flight is likely to be delayed”, “it is possible that I can pass the exam”, ...

• In applied mathematics, **probability** is the measure of the possibility that an event happens. The measure is between 0 and 1.

• **Probability** lies at the heart of electrical engineering and computer science:
  – communications
  – radar
  – machine learning
  – artificial intelligence
  – data mining
  – speech and image processing
  – ...

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Sentence Completion
Classification

- Classification is one of the most basic tasks in machine learning.

Training

Testing: am I a cat or dolphin?
Topic Mining

Topic 1: Clinton, White House, Scandal, Lewinsky, grand jury...

Topic 2: Utah, Chicago, NBA, Jordan, Carl, jazz, bull, basketball, final,...

Topic 3: NASA, Columbia, shuttle, space, experiments, ...
## Some Recent Results

Table 1: Mined topics from 5 classes of (1,683) articles of the TDT2 corpus.

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**References:**

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Why do we care about probability?

• In science and engineering, normally we only have partial knowledge of the outcome of a certain process or experiment.

• However, we do know that certain outcomes are “more likely” than others.

• Example:
  – **Experiment**: call my friend in Minneapolis, Minnesota, to ask if it is now sunny or snowy.
  – the possible outcomes:
    \[
    \begin{align*}
    \text{sunny with probability } & 0.2 \\
    \text{snowy with probability } & 0.8
    \end{align*}
    \]

  – But what if you are told that now it is sunny in St. Paul (10 miles away)?

• In this course, we will introduce a relatively sophisticated machinery to handle things like the above.
Experiments, sample space, events

• In probability, a basic model is *repeatable experiments*.

• An **experiment** consists of *procedure, observations, and model*.
  
  – **Exp**: flip a coin 3 times. **Observation**: the side you see at each time. **Model**: head and tail are with equal probabilities.
  – **Exp**: flip a coin 3 times. **Observation**: the number of heads that you see. **Model**: head and tail are with equal probabilities.

• An **outcome** of an experiment is any possible observation.

• The **sample space** of an experiment is the set of fine-grained, mutually exclusive, and collectively exhaustive elementary outcomes.

• An **event** is a subset of the sample space.
Experiments, sample space, events

- **Example 1**: You visit a gas station for 3 times and fill up your tank. You count the number of times that you pay over 50 dollars.

- The sample space (denoted by $S$) is

$$S = \{0, 1, 2, 3\}$$

- Some events:
  - $E_1$: you paid over 50 dollars at least twice: $E_1 = \{2, 3\}$.
  - $E_2$: you paid over 50 dollars exactly twice: $E_2 = \{2\}$. 
Experiments, sample space, events

- **Example 2**: You visit a gas station for 3 times and fill up your tank. You record the following at each time:

\[
\begin{cases}
0 & \text{if you paid less than $50} \\
1 & \text{if you paid over $50}
\end{cases}
\]

- The sample space is

\[ S = \{000, 001, 010, 100, 011, 101, 110, 111\} \]

- different designs (including definitions of observation) leads to different sample spaces.
Experiments, sample space, events

- **Example 3**: You visit a gas station for 2 times and fill up your tank. You record exactly how much you pay at each time.

- For the first time, the outcome takes a value $\in [30, 70]$, and the same for the second time.

- What does the sample space look like?
**Example 3**: You visit a gas station for 2 times and fill up your tank. You record exactly how much you pay at each time.

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Experiments, sample space, events

- **Example 3**: You visit a gas station for 2 times and fill up your tank. You record exactly how much you pay at each time.

- For the first time, the outcome takes a value $\in [30, 70]$, and the same for the second time.

- Consider an event:

  \[ E_1 : \text{the first time you pay less than 38 and the second time less than 42.} \]
Experiments, sample space, events

- **Example 3**: You visit a gas station for 2 times and fill up your tank. You record exactly how much you pay at each time.

- \( E_1 \): the first time you pay less than 38 and the second time less than 42.
Experiments, sample space, events

• **Example 3**: You visit a gas station for 2 times and fill up your tank. You record exactly how much you pay at each time.

• \(E_1\): the first time you pay less than 38 and the second time less than 42.

Remark: Examples 1-2 are related to **discrete random variables** and example 3 **continuous random variables**; we’ll cover both cases.