Q1. Prove the following facts:

(a) \( P[A \cup B] \geq P[A] \). (5%)
(b) \( P[A \cup B] \geq P[B] \). (5%)
(c) \( P[A \cap B] \leq P[A] \). (5%)
(d) \( P[A \cap B] \leq P[B] \). (5%)

Q2. In an experiment, \( A, B, C \) and \( D \) are events with probabilities \( P[A] = 1/4 \), \( P[B] = 1/8 \), \( P[C] = 5/8 \), and \( P[D] = 3/8 \). Furthermore, \( A \) and \( B \) are disjoint, while \( C \) and \( D \) are independent.

(a) Find \( P[A \cap B] \), \( P[A \cup B] \), \( P[A \cap B^c] \), and \( P[A \cup B^c] \). (5%)
(b) Are \( A \) and \( B \) independent? (5%)
(c) Find \( P[C \cap D] \), \( P[C \cap D^c] \), and \( P[C^c \cap D] \). (5%)
(d) Are \( C^c \) and \( D^c \) independent? (5%)

Q3. For independent events \( A \) and \( B \), prove that

(a) \( A \) and \( B^c \) are independent. (10%)
(b) \( A^c \) and \( B \) are independent (5%)
(c) \( A^c \) and \( B^c \) are independent (5%)
Q4. Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or a negative (-) response. Suppose the test gives the correct answer 99% of the time. What is $P[-|H]$, the conditional probability that a person tests negative given that the person does have the HIV virus? What is $P[H|+]$, the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive? (20%)

Q5. Answer the following questions:

(a) Prove that $P[A \cup B] = P[A] + P[B] - P[A \cap B]$ for any $A$ and $B$ (not necessarily disjoint). (10%)

(b) Now generalize: find a formula for $P[A \cup B \cup C]$ for any $A, B, C$ (not necessarily mutually exclusive). (10%)