Structured Models II

- Discuss transient and asymptotic dynamics
- Define sensitivities, elasticities, and review their uses
- Explore an extended example with some additional analyses of a simple model if there is time!

Asymptotic and Transient Dynamics

Asymptotic:
- Function of matrix entries
- NOT affected by start point
- Also known as ergodic behavior

Transient:
- A function of start vector
- NO effect on long-term dynamics
- Transient period determined by matrix elements
- Deterministic, time-invariant entries

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Problem 3e on page 64 of Case:

The fictitious field shrew

_Hopefully, you noticed that even after eight time steps, this 3 x 3 model had failed to converge to a stable stage distribution…_ 

Excel Demo!

Eigenvalues and Eigenvectors

All biologically reasonable matrices of n stages will have n eigenvalues.

There will be a dominant eigenvalue

There will be a left and right eigenvector associated with that eigenvalue

Mathematical meaning and biological interpretation on the board…

Sensitivity and Elasticity Analysis

Used to explore system behavior

_Sensitivity: which model entry has the greatest influence, e.g. on growth rate?_

Matrix models have analytical solution using linear algebra

Iterative methods possible on ANY model

Sensitivity

\[
\frac{\partial \lambda}{\partial a_{ij}} = \frac{\mathbf{v}_i \mathbf{w}_j}{\langle \mathbf{w}, \mathbf{v} \rangle}
\]

“the rate of change in lambda relative to the rate of change in matrix entry \(a_{ij}\) is equal to the product of the \(i\)th and \(j\)th entries of the eigenvectors divided by the eigenvector dot product.”
**Elasticity**

Elasticity is the proportional change in lambda per proportional change in the matrix element.

\[ e_{ij} = \frac{a_{ij}}{\lambda} \times \frac{\partial \lambda}{\partial a_{ij}} \]

Example: Another exel demo!!

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Matrix entries and underlying demographic rates...

Note that formulas are for \( a_{ij} \)...

But, these entries may be composed of more than one vital rate: \( F=S_0b_x \)

Can calculate sensitivities and elasticities for underlying demographic rates

*Elasticities of underlying rates will not sum to unity! Watch out!*

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**Elasticity and Sensitivity, Summary**

Use with caution
Interpret with care
Not a substitute for thoughtful, biologically informed management!

*Still, very useful especially in situations where little data are available...*

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**If more than one year of data exist...**

Demographic data over time or a range of conditions
Demographic data for a series of treatments

*What else can we do besides look at transient and asymptotic dynamics for each set of demographic rates?*

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**Retrospective versus Prospective Analysis**

So far, we have projected our models forward in time- what would happen if nothing changed?

This is formally known as a prospective analysis
Comparison of scenarios- “What if...?”

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**Example:**

Burrowing owls’ dynamics are affected by both vole population cycles and pesticides...

- 4 years of demographic data
- Years included a peak and crash event for California vole populations
- Evidence that food shortage combined with low \( p,p' \)DDE levels caused harm to reproduction
Prospective Analysis

\[ n_{(t+y)} = A_c A_p (A_a)^k n_{(t)} \]

- Peak year always followed by crash year
- Peak-crash followed by 1-10 average years
- For each cycle frequency, reproduction was reduced by up to 50% during crash year

*Calculated long-term \( \lambda \) and sensitivities of \( \lambda \) to demographic parameters for each scenario*

Retrospective Analysis

Suppose we have a series of matrices from demographic rates estimated over a range of conditions or years...

What changes in demographic rates led to the changes in population growth rate from year to year?

Also known as a “Life Table Response Experiment”, or LTRE.
Life Table Response “Experiment”

\[ \lambda^{(m)} - \lambda^{(a)} = \sum (\theta^{(m)} - \theta^{(a)}) \frac{\partial \lambda}{\partial \theta} \bigg|_{(B_m + B_a)/2} \]

*Which parameter most influences the changes in population growth compared to an “average” year?*

- Peak year population growth almost entirely due to increased reproductive output
- Bad year population growth due to declines in adult (and juvenile) survival

Gervais, Hunter, and Anthony. Ecological Applications, in press.

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Incorporating Variability

- Use more than one set of matrix values
  a periodic projection model-
- Incorporate uncertainty into matrix entries
  rates are means and variances, so the model is no longer deterministic
- Matrix entries can also be equations

*Nonlinear dynamics, density dependence*