ECON 463/563: Farrell Decompositions of Efficiency

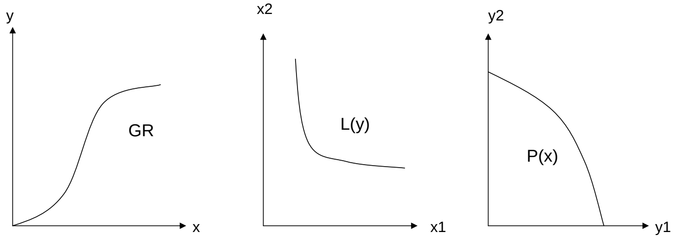
based on OnFront, Reference Guide
Coelli, et al., *Introduction to Efficiency and Productivity*
Springer, 2005, Ch 6.

April 22, 2008

Outline

- Decomposition of CRS technical efficiency into Scale Efficiency and VRS technical efficiency
- Decomposition of cost efficiency
- Decomposition of revenue efficiency

For these we will use all of our equivalent technologies



GR={(x,y):x can produce y}

L(y)={x: (x,y) is in GR}

P(x)={y: (x,y) is in GR}

If (x,y) is in GR, then x is in L(y) and y is in P(x)

Decomposition of CRS technical efficiency into Scale Efficiency and VRS technical efficiency

Input-based: $F_i(y, x|C, S) = F_i(y, x|V, S) \cdot SE_i$

Output-based: $F_o(x, y|C, S) = F_o(x, y|V, S) \cdot SE_o$

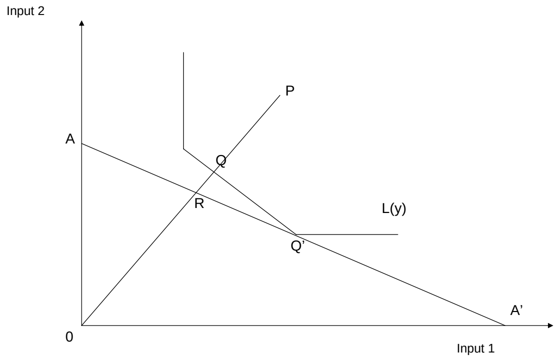
Recall: $SE_i = F_i(y, x|C, S)/F_i(y, x|V, S)$

Rearranging this definition gives us the $F_i(y, x|C, S)$ decomposition. Similarly for SE_o.

Decomposing Cost Efficiency: Farrell (1957)

$$\text{Cost effic} = \text{technical efficiency} \cdot \text{allocative efficiency}$$

$$OR/OP = OQ/OP \cdot OR/OQ$$



Note:

We use the input set $L(y)$ as our technology for cost efficiency.

Recall:
cost $= w_1 \cdot x_1 + w_2 \cdot x_2$
thus the slope of the isocost curves is $-w_1/w_2$
since $x_2 = cost/w_2 - w_1/w_2 \cdot x_1$.

We could include additional isocosts through the observed input bundle and the technically efficient bundles, these would be parallel to the minimum isocost in the figure.

$$\begin{aligned}\text{Overall Cost Efficiency: } O_i(y,x,w) &= OR/OP \\ &= \text{minimum cost/observed cost}\end{aligned}$$

$$\begin{aligned}\text{Technical Efficiency: } F_i(y,x) &= OQ/OP \\ &= \text{cost at Q/observed cost}\end{aligned}$$

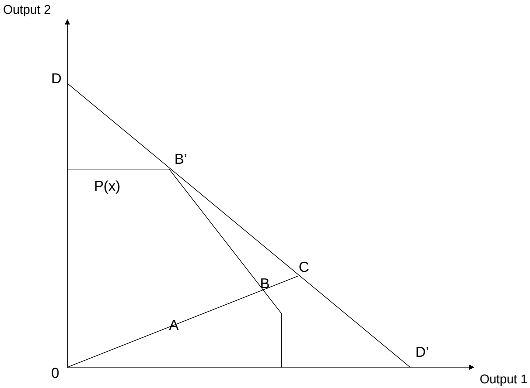
$$\begin{aligned}\text{Allocative Efficiency: } A_i(y,x,w) &= OR/OQ \\ &= \text{cost at R/cost at Q}\end{aligned}$$

$$\begin{aligned}O_i(y,x,w) &= F_i(y,x) \cdot A_i(y,x,w) \\ OR/OP &= OQ/OP \cdot OR/OQ\end{aligned}$$

Revenue Efficiency: Farrell (1957)

Overall Rev efficiency= Technical efficiency · Allocative efficiency

$$OC/OA = OB/OA \cdot OC/OB$$



Note:

We use the output set $P(x)$ as our technology for revenue efficiency.

Recall:
revenue $= p_1 \cdot y_1 + p_2 \cdot y_2$
thus the slope of the isorevenue curve in the figure is $-p_1/p_2$
since $y_2 = rev/p_2 - p_1/p_2 \cdot y_1$.

We could include additional isorevenues through the observed output bundle and the technically efficient bundles, these would be parallel to the maximum isorevenue in the figure.

$$\text{Overall revenue Efficiency: } O_o(y,x,w) = OC/OA$$
$$= \text{maximum rev/observed rev}$$

$$\text{Technical Efficiency: } F_o(y,x) = OB/OA$$
$$= \text{revenue at B/observed rev}$$

$$\text{Allocative Efficiency: } A_o(y,x,w) = OC/OB$$
$$= \text{rev at C/rev at B}$$

$$O_o(y,x,w) = F_o(y,x) \cdot A_o(y,x,w)$$
$$OC/OA = OB/OA \cdot OC/OB$$