Measuring Productivity

Econ 463/563

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Sources and Definitions

T.J. Coelli, D.S. Prasada Rao, C.J. O'Donnell and G.E. Battese An Introduction to Efficiency and Productivity Analysis, second edition, Springer, NY, 2005, 67-83, 289-310.

S. Grosskopf, Efficiency and Productivity, in H. Fried, C.A.K. Lovell and S. Schmidt, eds, The Measurement of Productive Efficiency, Oxford Univ. Press, 1993, 160-196.

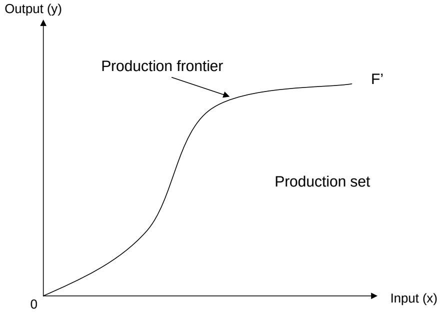
Some informal definitions

- feasible production set (technology)
- technical efficiency
- productivity
- total factor productivity (TFP)
- technical change

Feasible technology: single input and output

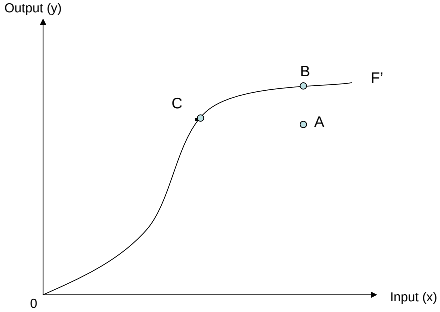
feasible technology set: 0F' and area underneath

production frontier: 0F'



Technical Efficiency

A firm (DMU) is **technically efficient** if it operates on the frontier of technology. A DMU is **technically inefficient** if it operates beneath the frontier. Which DMUs are technically efficient? Scale efficient?



Productivity

The **productivity level** of a firm (DMU) is the ratio of output(s) to input(s) it uses. For the single input single output case:

$$productivity = output/input = average\ product \tag{1}$$

An example of a single factor productivity index is labor productivity: output/labor.

Total Factor Productivity (TFP)

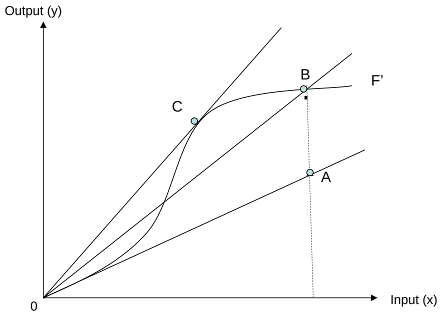
- **Total factor productivity (TFP)** is the ratio of an index of *all* outputs produced to an index of *all* inputs employed.
- With many outputs and many inputs we need to *aggregate* inputs and outputs to obtain indexes to form a ratio to measure total factor productivity.

Aggregation of Outputs and Inputs for TFP

Traditionally, economists have used prices or shares to aggregate inputs and outputs:

- Paasche
- Laspeyres
- Fisher
- Törnqvist

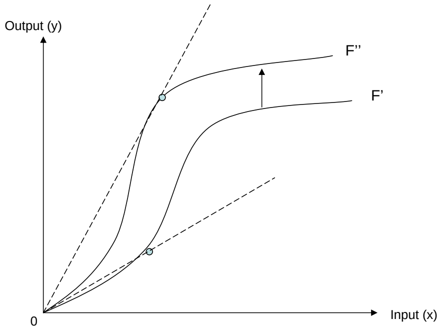
In contrast, the **Malmquist productivity index** does not require price data to aggregate inputs, rather it uses distance functions (the reciprocal of Farrell technical efficiency) to aggregate.



Which DMU has the highest productivity? Is it technically efficient? Is it scale efficient?

Productivity over time

Productivity growth: comparison of output/input over time. Note that the frontier can shift over time.



Simple Case: single input, single output, no inefficiency

Assume: two periods, t and t+1

$$TFP \text{ in per t+1} = y^{t+1}/x^{t+1} \text{ (ave product in t+1)}$$

$$TFP \text{ in per t} = y^{t}/x^{t} \text{ (ave product in t)}$$

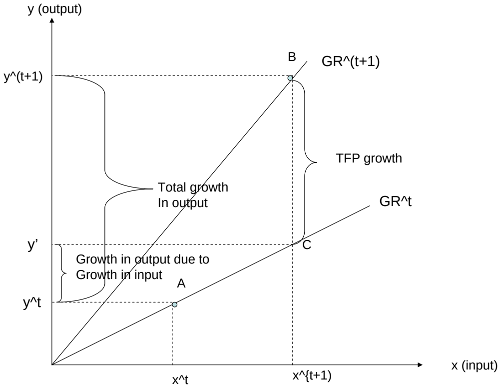
TFP Growth in ratio form:

$$\frac{TFP^{t+1}}{TFP^{t}} = \frac{y^{t+1}/x^{t+1}}{y^{t}/x^{t}} = \frac{y^{t+1}/y^{t}}{x^{t+1}/x^{t}}$$

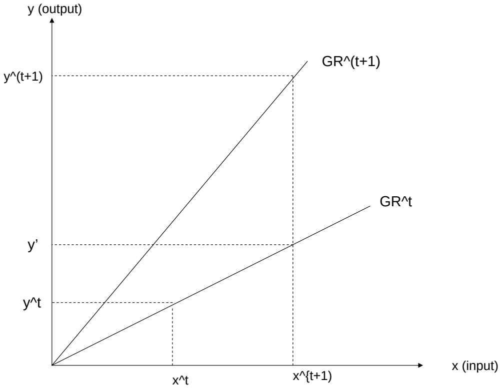
Rearranging:

$$(y^{t+1}/y^{t}) = (x^{t+1}/x^{t}) \cdot (TFP^{t+1}/TFP^{t})$$

How do we get from A to B?



$$(y^{t+1}/y^t) = (x^{t+1}/x^t) \cdot (TFP^{t+1}/TFP^t)$$



Growth in x $= (x^{t+1}/x^t) = (y'/y^t)$
Growth in TFP $= (y^{t+1}/y^t)/(y'/y^t) = (y^{t+1}/y')$
Growth in TFP $=$
$D_o^t(x^{t+1}, y^{t+1})/D_o^t(x^t, y^t) = F_o^t(x^t, y^t)/F_o^t(x^{t+1}, y^{t+1})$

Remarks

- With no technical inefficiency, TFP growth = technical change.
- Recall that technical change occurs when the frontier shifts.

TFP Growth in difference (residual) form:

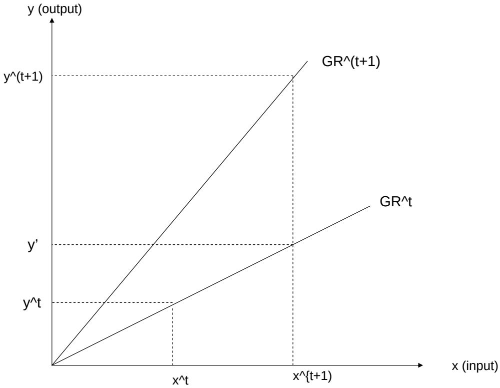
Growth in TFP= Growth in y - Growth in x

Rearranging:

Growth in y = Growth in x + Growth in TFP

or *approximately*

$$\begin{aligned} \Delta TFP &= \Delta y - \Delta x \\ \text{Rearranging: } \Delta y &= \Delta x + \Delta TFP \\ \text{or } (y^{t+1} - y^{t}) &= (x^{t+1} - x^{t}) + (TFP^{t+1} - TFP^{t}) \end{aligned}$$

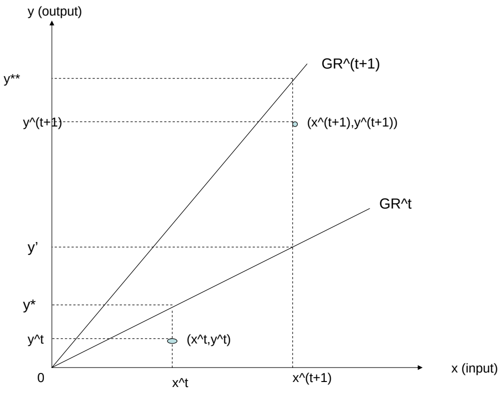


$$\Delta y = (y^{t+1} - y^t)$$

$$\Delta x = (x^{t+1} - x^t) = (y' - y^t)$$

$$\begin{aligned} \Delta TFP = \Delta y - \Delta x &= (y^{t+1} - y^t) - (y' - y^t) \\ &= y^{t+1} - y' \end{aligned}$$

Again, with no technical efficiency, productivity growth is identical to technical change, i.e., the shift in the frontier of technology. What happens if we have technical inefficiency?



$$
\begin{aligned}
\Delta y &= \mathit{eff}^{\,t} \cdot \Delta x \cdot \mathit{tech\ change} \cdot \mathit{eff}^{\,t+1} \\
(y^{t+1}/y^{t}) &= (y^{*}/y^{t}) \cdot (y'/y^{*}) \cdot (y^{**}/y') \cdot (y^{t+1}/y^{**}) \\
\Delta TFP &= \Delta y/\Delta x = \text{eff change} \cdot \text{tech change}.
\end{aligned}
$$

Malmquist Productivity Index

The Malmquist productivity index uses distance functions (reciprocals of our technical efficiency measures) to measure total factor productivity in the presence of inefficiency. Its major advantage is that no price data is required to 'aggregate' inputs and outputs. It also provides the decomposition of TFP growth into efficiency change and technical change:

$$\text{Malmquist prod change} = \text{effic change} \cdot \text{tech change}$$

Färe, Grosskopf, Lindgren and Roos (1989)(hereafter FGLR) defined the Malmquist productivity indexes for adjacent periods as

$$M_t^{t+1} = \left(\frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)} \frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \right)^{1/2}.$$

$$MEFFCH_t^{t+1} = \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)}$$

and

$$MTECH_t^{t+1} = \left[\frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)} \right]^{1/2}$$

where

$$M_t^{t+1} = MEFFCH_t^{t+1} \cdot MTECH_t^{t+1}.$$