Review of Production Theory

based on OnFront, Reference Guide
Coelli, et al., *Introduction to Efficiency and Productivity*
Springer, 2005, Ch 3.

April 9, 2008

Technology: Set Representations

Technology describes the relationship between inputs x and outputs y (denoted by q in Coelli et al), where

$$\begin{array}{rll} x_n & n = 1, \ldots, N & \text{n indexes different types of } \textbf{inputs} \\ x = & (x_1, \ldots, x_N) & \text{is the vector of inputs} \\ y_m, & m = 1, \ldots, M & \text{m indexes different types of } \textbf{outputs} \\ y = & (y_1, \ldots, y_M) & \text{is the vector of outputs} \end{array} \quad (1)$$

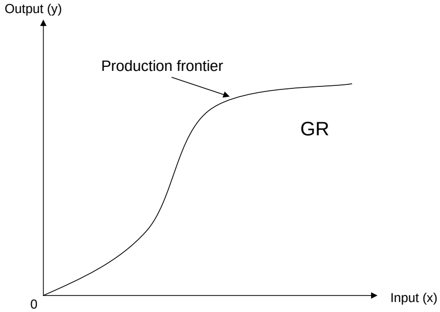
Equivalent Technology Sets:

- Input-Output Set: **GR** or **S** in Coelli et al.
  Set of all feasible input and output bundles.
- Output Set: **P(x)**
  Set of all outputs producible from x.
- Input Set **L(y)** (L(q) in Coelli et al): The set of all inputs that can produce output vector y

Input-Output Set

Set of all feasible input and output bundles.

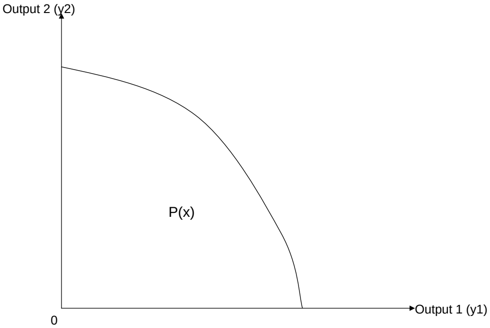
$$GR = \{(x, y) : x \text{ can produce } y\} \quad (2)$$



Output Set

Output Set **P(x)**: the set of all outputs producible from given inputs, x

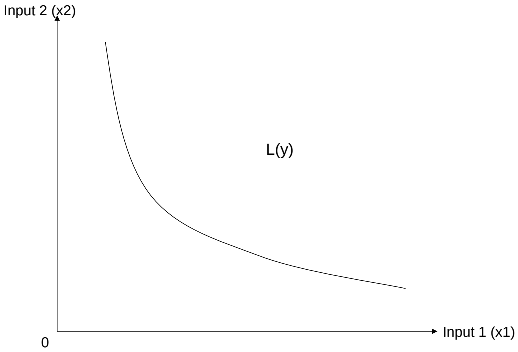
$$P(x) = \{(y) : (x, y) \text{ belongs to } GR\} \quad (3)$$



Input set

Input Set **L(y)** (L(q) in Coelli et al): The set of all inputs that can produce output vector y

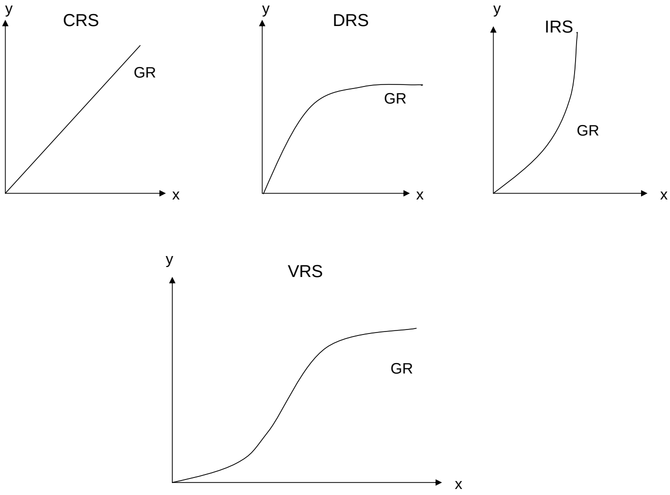
$$L(y) = \{(x) : (x, y) \text{ belongs to } GR\} \quad (4)$$



Returns to Scale: Input-Output Set

- Constant Returns to Scale (CRS)
  doubling inputs doubles outputs
- Decreasing Returns to Scale (DRS)
  doubling inputs yields less than double outputs
- Increasing Returns to Scale (IRS)
  doubling inputs yields more than double outputs
- Variable Returns to Scale (VRS) allows for IRS, CRS and DRS

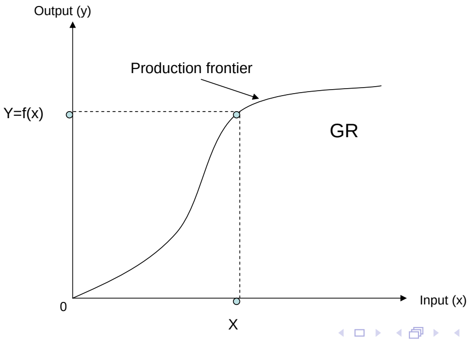
Returns to Scale



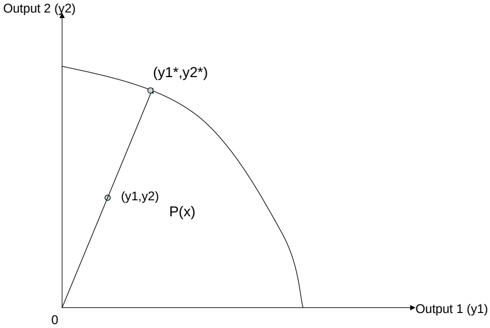
Function Representations of Technology

- **Production Function**: where y is a single output

$$f(x) = \max\{y : y \in P(x)\} \quad (5)$$

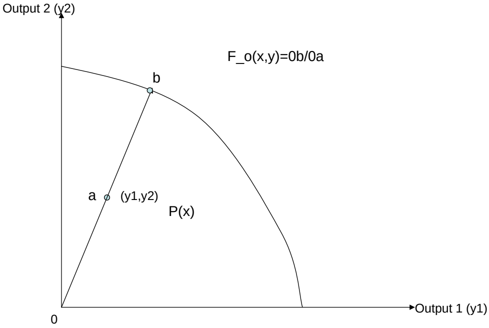


- **Distance Functions**: multiple inputs and outputs
- **Output Distance Function**
  $D_o(x, y) =$
  1 if y is technically efficient, i.e., on the boundary of P(x)
- If y is single-valued, then $D_o(x, y) = y/f(x)$.

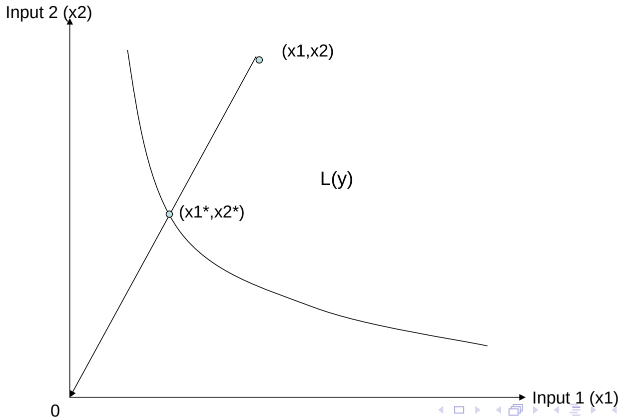


Farrell output based technical efficiency

$1/D_o(x, y)$ =Farrell output measure of technical efficiency, $F_o(x, y)$
$F_o(x, y) =$ maximal output/observed output

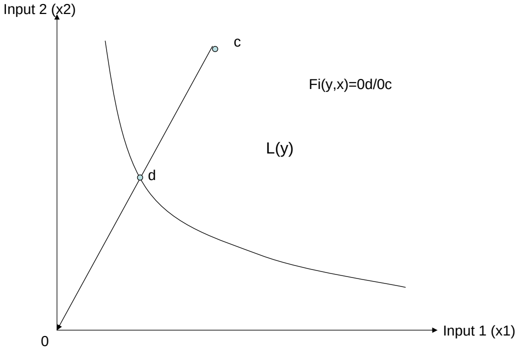


- **Input Distance Function**
  $D_i(y, x) = 1$ if x is technically efficient, i.e., on the boundary of L(y)
  $1/D_i(y, x) =$Farrell input measure of technical efficiency



Farrell input based technical efficiency

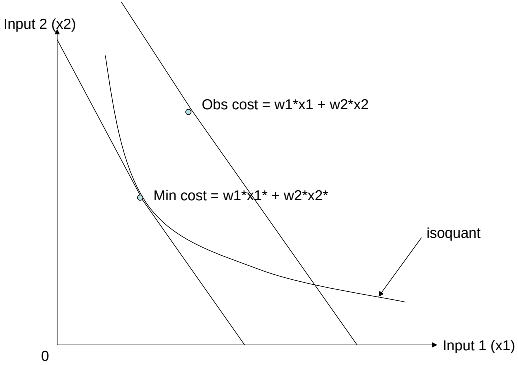
$F_i(y, x) =$ efficient input/observed input



Cost Function

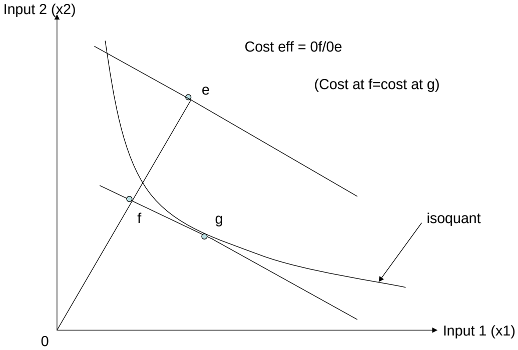
$$C(y, w) = \min_x \{wx : x \in L(y)\}, \tag{6}$$

where w is a vector of input prices.



Cost Efficiency

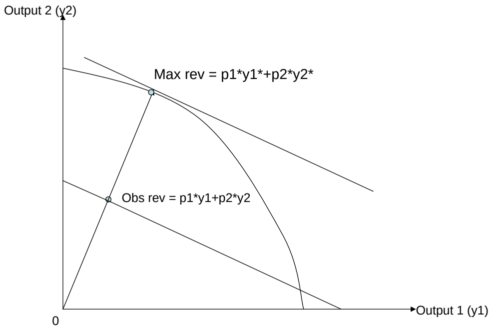
Cost efficiency = minimal cost/observed cost



Revenue Function

$$R(x, p) = \max_{y}\{py : y \in P(x)\}, \tag{7}$$

where p is a vector of output prices.



Profit Function

$$\Pi = \max_{x,y}\{py - wx : (x, y) \in GR\} \tag{8}$$