Solutions

Name:
MTH 355 Quiz 2

No notes nor calculators are allowed. THERE ARE MORE QUESTIONS ON THE BACK!

1. (1.00 points) Give examples of partitions of $\mathbb{Z}$ such that

(a) there is exactly one block

\[ \mathbb{Z} \]

(b) there are exactly two blocks

\[ \mathbb{Z}, \mathbb{Z} \setminus \{0\} \text{ and } \mathbb{Z}, \{0\} \]

(c) there are infinitely many blocks

\[ \mathbb{Z}, \mathbb{Z} \setminus \{0\} \]

(d) each block is a set of cardinality 2

\[ \mathbb{Z}, \{2n, 2n+1\} \]

2. (0.75 points) Decide if the following are true or false:

(a) $\forall x \in \mathbb{R}\setminus\{0\} \exists y \in \mathbb{R}, xy = 1$. True

(b) $\exists y \in \mathbb{R} \forall x \in \mathbb{R}\setminus\{0\}, xy = 1$. False

(c) $\exists b \in \mathbb{Q} \forall a \in \mathbb{Q}, ba = 0$. True
3. (0.75 points) Prove if $a$ is a rational number and $b$ is an irrational number, then $a + b$ is an irrational number.

Proof. By contradiction,
we assume $a$ is rational, $b$ is irrational, and $a + b$ is rational.
We can then take $m, n, p, q \in \mathbb{Z}$ with $n, q \neq 0$ such that $a = \frac{m}{n}$ and $a + b = \frac{p}{q}$.

But then
$$b = a + b - a = \frac{p}{q} - \frac{m}{n} = \frac{pn - mq}{qn}.$$ 
Now $pn - mq, qn \in \mathbb{Z}$ and $qn \neq 0$.
Thus $b$ is rational, contradicting the assumption that $b$ is irrational. $\square$