(1) True or False?

(a) If $f \circ g$ is one-to-one, then $g$ is one-to-one.

(b) If $f \circ g$ is onto, then $g$ is onto.

(c) The function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n^2$ is injective (1-1).

(d) The function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n^2$ is surjective (onto).

(e) The function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n^2$ is bijective.

(f) A connected graph on $n$ vertices must have at least $n - 1$ edges.

(g) The sum of all the vertex degrees in a graph is even.

(h) Every Euler cycle is a Hamiltonian cycle.

(i) There exists a graph whose vertices have the following degrees: 2, 2, 1, 1, 1, 1.

(j) There are 60 rearrangements of the word *APPLE*. 
(2) How many 5-card deals from a standard deck of 52 contain at least one card from each of the four suits?

(3) How many onto functions are there from a set with 4 elements to a set with 3 elements?
(4) Suppose 8 fish are to be distributed in 3 distinct tanks. In how many ways can this be done if...

(a) ...the fish are distinguishable?

(b) ...the fish are indistinguishable?

(c) ...the fish are indistinguishable and each bucket must get a fish?

(5) Assume $f : \mathbb{Z} \to A$ given by $f(x) = 2x - 5$ is onto. Determine the set $A$.

(6) Determine if $g : \mathbb{R} \to (0, 1]$ given by $f(x) = \frac{1}{1+x^2}$ is an injection (1-1), a surjection (onto), and a bijection.

(7) Your friend says that his “favorite (simple) graph” has the following degrees: 1, 1, 1, 2, 3, 4, 4, 5. Does such a graph exist? If so, draw it. If not, justify.

(8) Show that if a graph has exactly two vertices $v, w$ of odd degree than there is a path between $v$ and $w$. Hint: Suppose there isn’t a path between $v$ and $w$. Consider the connected component of $v$. This subgraph has a degree sum that is...

(9) Is the binary relation $\{(1, 1), (2, 3), (3, 2), (3, 3), (4, 4), (5, 6), (6, 6)\}$ on $\mathbb{N}$ symmetric?

(10) Is the binary relation $\{(2, 2), (2, 3), (3, 3), (3, 4), (4, 4), (5, 6), (5, 7), (6, 7)\}$ on $\mathbb{N}$ anti-symmetric?
(11) Let $f : \mathbb{N} \to \mathbb{N}$ be given by $f(1) = 1$ and for $n \geq 2$, $f(n) = n - 1$. Prove that $f$ is a surjection (onto) but not a bijection.

(12) Let $f : \mathbb{N} \to \mathbb{N}$ be given by $f(n) = n + 1$. Prove that $f$ is an injection (1-1) but not a bijection.

(13) Let $X = \{0, 1, 2, 3, 4\}$. Let $R$ be a relation on $X$ given by $(a, b) \in R$ if $(b - a) \mod 3 = 0$.

(a) Show $R$ is an equivalence relation.

(b) Find $A$, the matrix of the relation $R$. Use the standard numerical order on $X$ to index the rows and columns.
(14) Consider the binary relation $R$ on $\mathbb{R}$ given by $(x, y) \in R$ if $x^2 = y^2$. Determine if this relation is reflexive, symmetric, transitive, and anti-symmetric. Is $R$ a partial order? Is $R$ an equivalence relation?

(15) Consider the binary relation $R$ on $\mathbb{N}$ given by $(m, n) \in R$ if $m$ divides $n$. Determine if this relation is reflexive, symmetric, transitive, and anti-symmetric. Is $R$ a partial order? Is $R$ an equivalence relation?

(16) Consider the binary relation $R$ on the set of all people given by $(p, q) \in R$ if person $p$ is a child of person $q$. Determine if this relation is reflexive, symmetric, transitive, and anti-symmetric. Is $R$ a partial order? Is $R$ an equivalence relation?

(17) Prove that if $R$ and $S$ are both symmetric binary relations on $X$ then $R \cup S$ is a symmetric binary relation on $X$. 
(18) Draw a graph such that...

(a) ...there is a Hamiltonian cycle but no Euler cycle.

(b) There is an Euler cycle but no Hamiltonian cycle.

(19) How many non-negative integer solutions are there to \( x_1 + x_2 + x_3 = 20 \)?
Apply Dijkstra’s algorithm to find the length of the shortest path between vertices $u, v$ in the graph. Is the path unique?
(21) Apply Kruskal’s algorithm to find a minimal spanning tree in the graph.

(22) Review chapters 1 and 2, including the midterm review problems and the midterm!