(1) Which of the following are propositions? If it is a proposition, determine its truth value:

(a) Is Jack 20 years old?

(b) Do your homework!

(c) This statement is not a proposition.

(d) $x < 2$.

(e) If $3 < 2$ then $\sqrt{-1} \in \mathbb{R}$.

(f) For all finite sets $X$ and $Y$, it follows that $|X \times Y| = |X||Y|$. 

(g) Does induction require the base case(s)?

(h) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y^4 = x$.

(i) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, y^4 > x$.

(j) $\exists y \in \mathbb{Q}, \forall x \in \mathbb{R}, y < x + 10$.

(k) $\exists y \in \mathbb{Q}, \forall x \in \mathbb{R}, y < x^2$. 

WARNING: This is not a “sample test.” Problems on the midterm may or may not be similar to these problems. These problems are just intended to focus your study of the topics to appear on the midterm.
(2) Let $U = \{n \in \mathbb{Z} | 0 \leq n \leq 9\}$. Let $A = \{2, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$.

(a) Find $A - B$.

(b) Find $\overline{A \cup B}$.

(c) How many elements does the power set $\mathcal{P}(A)$ have?

(3) Assume $p$ and $q$ are true propositions, while $r$ is a false propositions. Find the truth value of the following propositions.

(a) $\neg(p \wedge q) \lor (p \rightarrow \neg(q \wedge r))$.

(b) $((p \wedge q) \rightarrow r) \wedge (p \wedge q)$.
(4) Let the domain of discourse be the real numbers. Determine the truth value of the following quantified propositions.

(a) \( \forall x, (x < 2 \rightarrow x^2 < 4) \).

(b) \( \exists x, (x < 2 \rightarrow x^2 < 4) \).

(5) Let the domain of discourse be the direct product of the real numbers with the real numbers. Determine the truth value of the following quantified propositions.

(a) \( \exists x, \forall y, x^2 + y < 0 \).

(b) \( \forall x, \exists y, x^2 + y < 0 \).

(6) Negate the quantified proposition in Problem (5), Part (a).
(7) Prove the following propositions or find a counterexample.

(a) If \( m \) is an even integer and \( n \) is an odd integer then \( mn \) is even.

(b) Let \( m, n \) be integers. If \( mn \) is an even integer then \( m \) and \( n \) are even integers.

(8) Use induction to prove the following proposition:

For all integers \( n \geq 4 \), \( n^2 \leq 2^n \).
Use induction to prove the following proposition:

For all integers, \( n \geq 1, \)

\[
1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{4n^3 - n}{3}.
\]
(10) Prove the following proposition or find a counterexample:

For all sets $A$, $B$ and $C$, if $A \times C = B \times C$ then $A = B$.

(11) Prove the following proposition or find a counterexample:

For all sets $A$, $B$ and $C$, if $C \neq \emptyset$ and $A \times C = B \times C$ then $A = B$. 