(1) (20 points) Evaluate the following limits.

(a) 
\[
\lim_{x \to -1} \frac{1}{x^5} + 1 \quad \frac{1}{x + 1}.
\]

(b) 
\[
\lim_{x \to -\infty} \frac{5 - x^2}{x^2 - \sqrt{4x^4} + 1}.
\]
(c) \[
\lim_{x \to 3^+} \frac{1}{x^4 - 81}.
\]

(d) \[
\lim_{x \to 0} \left( \frac{3 - x^2}{3} \right)^{\frac{1}{x^2}}.
\]

Hint: Use \( \ln \lim = \lim \ln \).
(2) (15 points) Find the coordinates of the absolute minimum and maximum for each function on the interval $[0, 2]$:

(a) 
\[ f(x) = \frac{x + 1}{x - 1}. \]

(b) 
\[ g(x) = 1 + x - x^2. \]

(c) 
\[ h(x) = xe^{-x}. \]
(3) (20 points) Find \( \frac{dy}{dx} \).

(a) 
\[ y = \sqrt{1 + e^{\sqrt{x}}}. \]

(b) 
\[ y = \frac{x^2 - 3x + 2}{x - 2}. \]
(c) \[ y = \sin^{-1} x^{-2}. \]

(d) \[ x + 1 = \tan xy. \]
(4) (15 points) Suppose we know that the derivative of \( f(x) \) is

\[
    f'(x) = \frac{3}{(x-2)^2}.
\]

On what interval(s) is \( f(x) \) concave down?

(5) (25 points) You are building a large custom window in the shape of a semicircle atop a rectangle. The customer only wants framing on the perimeter of this shape and only wants to pay for 18 meters of framing material. What dimensions will let in the most light?

Hint: Maximize area subject to the perimeter requirement.
(6) (20 points) Consider the function

\[ f(t) = \frac{1000}{1 + 5e^{-4t}}. \]

(a) Find

\[ \lim_{t \to \infty} f(t). \]

(b) Find \( f'(t) \). Determine interval(s) of increase and decrease for \( f(t) \).

(c) Find \( f''(t) \).

(d) If this function models the growth of a population of kangaroos where \( t \) represents years after initial observation, when is the population growing most rapidly? Justify!
(7) (15 points) Suppose \( g(-1) = 1, g'(-1) = -5, f(1) = 3 \) and \( f'(1) = -2 \). Find an equation for the tangent line to \( y = h(x) \) given that \( h(x) = f(g(x)) \).

(8) (20 points) Air is being pumped into a spherical balloon at a rate of 5 cm\(^3\)/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

Volume of a sphere:

\[
V = \frac{4}{3} \pi r^3.
\]