Math 251 - Midterm 1

No graphing calculators are allowed. You may have a 3” by 5” handwritten note card.

Your section is determined by your TA:
Armentrout - 001       Kim - 002     Rode - 003   Wichitsongkram - 004

(1) (7 pts) On my answer sheet I “bubbled” the circles for the following: my name, a section number and a form number.

   (a) True,

   (2) (7 pts) Find

   \[ \lim_{{x \to -1}} \frac{-9x^2}{{\sqrt{x + 10}}}. \]

   (d) -3.

   (3) (7 pts) Find

   \[ \lim_{{x \to 4}} \frac{2 - \sqrt{x}}{x - 4}. \]

   (b) \(-\frac{1}{4}\),

(4) (7 pts) Find
\[
\lim_{x \to \pi^-} \cot x.
\]
(b) \(-\infty\),

(5) (7 pts) Find
\[
\lim_{x \to -\infty} \frac{3x - 7}{x + \sqrt{4x^2 + 1}}.
\]
(a) \(-3\),

(6) (7 pts) Find
\[
\lim_{x \to \infty} \frac{5x^3 + 4x - 11}{x^4 + 7x^2 - 8}.
\]
(b) 0,

(7) (7 pts) Find
\[
\lim_{x \to 1} \frac{x^2 - 1}{x^2 - 2x + 1}.
\]
(d) d.n.e.
(8) (7 pts) Which of the statements are true for the function $f(x)$ with the following graph?

![Graph of a function](image)

(d) All of the above.

(9) (7 pts) For what value of $c$ is the function,

$$f(x) = \begin{cases} 
  x^2 + c & \text{if } x \leq 2 \\
  cx + 1 & \text{if } x > 2
\end{cases}$$

continuous?

(d) 3.

(10) (7 pts) Which of the following will correctly calculate $f'(3)$ for $f(x) = \sqrt{x + 1}$?

(c) \[ \lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h}, \]
(11) (10 pts) Use a **limit definition of the derivative** to compute \( f'(1) \) for the function

\[
f(x) = \frac{1}{\sqrt{x}}.
\]

(12) (10 pts) Let

\[
f(x) = \frac{|x^2 - 9|}{x^2 + 3x}.
\]

Find the limits at \( \pm \infty \) to determine horizontal asymptotes. Find any vertical asymptotes. Find the one-sided limits on each side of any vertical asymptotes.
(13) (10 pts) Let

\[ f(x) = e^{\frac{1}{x}}. \]

Find both:

\[ \lim_{x \to 0^-} f(x) \] and \[ \lim_{x \to 0^+} f(x). \]
(14) (10 pts - Extra Credit) Prove using the precise definition:

\[ \lim_{{x \to 1}} (x^2 + x + 2) = 4. \]