Math 251  Limits !? !??  Activity 2 - Part I

This activity comes in TWO PARTS (the second of which is only released in recitation) and is worth 2\% of course credit and graded out of “10” points (5 points for completion, 5 points for correctness on a randomly chosen subset of the exercises). See tentative calendar on the syllabus for due dates. Late activities are accepted up to a day late with a 50\% penalty.

Whenever a box is provided, put your final answer for that part of the exercise in the box.

(1) Limits are fundamental concept for calculus. Describe in your own words what...

(a) \( \lim_{x \to a} f(x) = L \) tells us about \( f(x) \).

(b) \( \lim_{x \to a^+} f(x) = L \) tells us about \( f(x) \).

(c) \( \lim_{x \to a^-} f(x) = L \) tells us about \( f(x) \).
(2) Draw a graph of a function $f(x)$ with a domain of $[-10, 10]$ such that $\lim_{x \to 5^-} f(x) = 2$ while $\lim_{x \to 5^+} f(x)$ does not exist (as a finite number).

(3) Alshon and Marie are arguing about a pair of functions, $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$, and $g(x) = \frac{x^2 - 1}{x - 1}$. Alshon says $f(x) = g(x)$ because they have the same values everywhere since the simplified form of $g(x)$ is $x + 1$, which is 2 at $x = 1$. Marie says that $f(x) \neq g(x)$ because their domains are different as $g(1)$ is undefined. Whose right? Explain!
(4) Consider each limit. Determine the value (if it exists) or write “dne.”

(a) \( \lim_{x \to 2} \frac{x - 1}{x^2 + 4} \)

(b) \( \lim_{x \to 2} \frac{x + 1}{x^2 - 4} \)

(c) \( \lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3} \)

(d) \( \lim_{x \to 0} \sec (x) \)

(e) \( \lim_{x \to \pi} \frac{1 - \cos (x)}{\sin^2 (x)} \)

(f) \( \lim_{x \to 0} \sin (x^{-2}) \)
(5) Effusion in chemistry is the process in which two or more particles are escaping through a small hole with the lighter particles leaving the container faster than the heavy particles. Heavier particles move slower than light particles with the same kinetic energy giving lighter particles a higher probability of escaping every second. This relationship can be determined by the equation: 

$$r_e = \frac{k}{\sqrt{m}}$$

where \(m\) is the mass and \(k > 0\) is a constant.

(a) Show that the proportion of gas \(A\) escaping to gas \(B\) escaping (when equal numbers of particles are present) is

$$\frac{\sqrt{mm_2}}{\sqrt{mm_1}}$$

where \(mm_1\) is the mass of 1 mole of gas \(A\) and \(mm_2\) is the mass of one mole of gas \(B\) (called “molar masses”) measured in the same units.

(b) If a balloon is filled with equal numbers of Argon atoms and Hydrogen atoms then immediately after poking a small hole, what is the proportion of the rate of escaping Hydrogen atoms to the rate of escaping Argon atoms?