Math 251  Derivatives  Activity 4 - Part I

This activity comes in TWO PARTS (the second of which is only released in recitation) and is worth 2% of course credit and graded out of “10” points (5 points for completion, 5 points for correctness on a randomly chosen subset of the exercises). See tentative calendar on the syllabus for due dates. Late activities are accepted up to a day late with a 50% penalty.

Whenever a box is provided, put your final answer for that part of the exercise in the box.

(1) Given the following graph of $f(x)$, draw its derivative.
(2) Consider the following graph of $y = f(x)$ (solid curve) and $y = g(x)$ (dashed curve).

(a) Determine which function is the derivative of the other.

(i) $f'(x) = g(x)$

(ii) $g'(x) = f(x)$.

(b) Find an equation to the tangent line to $y = f(x)$ at $x = 2$. 


(3) Suppose \( f \) is differentiable with \( f'(3) = 2 \) and \( f'(3) = -3 \). Suppose that \( g(x) = x^2 f(x) \). What is the rate of change in \( g \) when \( x = 3 \)?

(4) Determine \( \frac{d}{dx} \left[ \frac{2f(x) + 1}{xg(x)} \right] \) at \( x = 2 \) given the following values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>-7</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
In chemistry it is important to know the speed of a molecule, which can be calculated using the “root mean square” equation. Using standard temperature (273 Kelvin), it is possible to find the velocity of molecules or elements with ranging mass. This is important because it determines multiple characteristics of a reaction. The equation is

\[ \nu = \sqrt{\frac{3RT}{M/1000}}. \]

where \( R \) is a constant, \( T \) is the temperature (here assumed to be constantly 273 K) and \( M \) is the molar mass of the molecule in grams. Here is the graph of \( \nu \) as a function of \( M \):

(a) Find \( \frac{d\nu}{dM} \).

(b) Find and interpret the value of \( \frac{d\nu}{dM} \) when \( M = 20 \) grams. Use \( T = 273 \) K and 
\( R = 8.31 \text{ kg m}^2 \text{ K}^{-1} \text{ mol}^{-1} \text{ sec}^{-2} \).