Math 251  The Chain Rule and Implicit Differentiation  Activity 5 - Part I

This activity comes in TWO PARTS (the second of which is only released in recitation) and is worth 2% of course credit and graded out of “10” points (5 points for completion, 5 points for correctness on a randomly chosen subset of the exercises). See tentative calendar on the syllabus for due dates. Late activities are accepted up to a day late with a 50% penalty.

Whenever a box is provided, put your final answer for that part of the exercise in the box.

(1) Suppose \( f(x) \) and \( g(x) \) are differentiable functions. Given that \( y = 2x - 3 \) is the equation of the tangent line to \( y = f(x) \) at \( x = 2 \) and \( y = -x + 3 \) is the equation of the tangent line to \( y = g(x) \) at \( x = 1 \), find the equation to the tangent line to \( y = f(g(x)) \) at \( x = 1 \) or justify that there isn’t enough information to determine it.

(2) Derive a formula for \( \frac{d^2}{dx^2} (f(g(x))) \) by using the Chain Rule and the Product Rule.
The amount of Vitamin D an individual produces varies depending on their age, regional conditions, skin pigmentation and exposure to sunlight. UV radiation in sunlight is necessary to produce the active form of Vitamin D which the body then uses to absorb calcium from the diet. Using the equation $D(t) = 33 - 15 \cos \left( \frac{\pi(t-1)}{6} \right)$ to represent the hypothetical average concentration of Vitamin D in a Corvallis resident throughout the year ($t$ is months into the year), answer the following questions.

(a) When do you think blood Vitamin D levels will be highest? Lowest? When do you think they will be increasing the most rapidly? Decreasing most rapidly?

(b) Find $D'(t)$.

(c) What does the first derivative tell us about Vitamin D concentration?

(d) When is the first derivative positive? When is it be negative?
(4) Consider the curve given by $\sqrt{y} + xy = 1$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

(b) Find the equation of the tangent line to the curve at the point on the curve where $y = 1/4$. 
(5) Find \( \frac{dy}{dx} \) where \( 3(x^2 + y^2)^2 = 25(x^2 - y^2) \).

(6) Find \( \frac{d^2 y}{dx^2} \) where \( x^2 + y^2 = 1 \).
(7) Suppose brain weight $B$ (in grams) as a function of body weight $W$ (in grams) in fish can be modeled by $B = 0.007W^{2/3}$. Another model for body weight as a function of body length $L$ (in cm) is $W = 0.12L^{5/2}$. Find $\frac{dB}{dL}$ as a function of $L$ and interpret its meaning.