Math 251  What derivatives tell us!  Activity 7 - Part I

This activity comes in TWO PARTS (the second of which is only released in recitation) and is worth 2% of course credit and graded out of “10” points (5 points for completion, 5 points for correctness on a randomly chosen subset of the exercises). See tentative calendar on the syllabus for due dates. Late activities are accepted up to a day late with a 50% penalty.

Whenever a box is provided, put your final answer for that part of the exercise in the box.

(1) For each function, find all critical points and classify them according to either the First Derivative Test or the Second Derivative Test.
   (a) $f(x) = 2x + \frac{1}{x}$

   (b) $f(x) = \frac{x}{x^2+1}$

   (c) $f(x) = \frac{x^2}{\sqrt{1-x^2}}$

   Hint: Pay attention to the domain!
(2) Suppose a biologist begin observing a population of beavers in a particular ecosystem over several years. Suppose that the biologists finds that \( P(t) = \frac{200}{1 + 9e^{-t/2}} \) is a good fit for the numbers of beavers in the population at \( t \) years (since the observations began).

(a) Find the growth rate \( P'(t) \) and interpret its meaning.

(b) When is the growth rate at a maximum?

(c) What will the population of beavers approach in the “long-run” in this ecosystem? What happens to the growth rate?
Glucose is a simple sugar that serves as an energy source in organisms. Scientists have determined that glucose absorption from the gastrointestinal tract (GI tract) in rats and rabbits can be modeled by the function $P(t) = 1 - e^{-at}$ where $P$ is the fraction absorbed (meaning when $P = 1$, all glucose has been completely absorbed), $t \geq 0$ is time (in hours) and $a > 0$ is a constant.

(a) Find $\frac{dP}{dt}$ and express it in terms of $a$ and $P(t)$.

(b) Find $\frac{d^2P}{dt^2}$ and express it in terms of $a$ and $P(t)$.

(c) Determine the intervals where $y = P(t)$ is concave up and concave down, and explain what it means in the context of the model. Does it make sense?
(4) Consider the function \( f(x) = \frac{\ln(x)}{x} \) where \( x > 0 \).

(a) Find the maximum value of \( f(x) \) and where it occurs.

(b) Using the above result, prove that \( e^\pi > \pi^e \).

Hint: At some point use that \( r \ln(x) = \ln(x^r) \).