Math 252  Improper Integrals and Separable ODEs  Activity 7

This activity is worth 10 points of course credit. See tentative calendar for due dates. Late activities are accepted at the discretion of your recitation instructor and a penalty may be imposed.

(1) Let’s test your intuition. Is the following statement true or false? Explain (with complete and grammatically correct sentences) your reasoning.

If an unbounded solid object has finite volume then any cross-section of the object must have finite area.

Note: Credit will be given for any coherent answer (regardless of whether it’s correct).
(2) Gabriel’s Horn! Consider the unbounded region $R$ under the curve $y = \frac{1}{x}$ (and above the $x$-axis) on the interval $[1, \infty)$.

(a) Show via an improper integral that the area of $R$ is infinite.

(b) Show via an improper integral that the volume of the solid obtained by revolving this region about the $x$-axis is finite and report the volume of this unbounded solid object.

$V =$
(3) For what value(s) of $p > 0$ is $\int_1^\infty \frac{1}{x^p} \, dx$ finite? For those value(s), determine the integral (as a function of $p$).

\[
\int_1^\infty \frac{1}{x^p} \, dx =
\]
(4) For what value(s) of $p > 0$ is $\int_0^1 \frac{1}{x^p} \, dx$ finite? For those value(s), determine the integral (as a function of $p$).

For the value(s) of $p$ above, $\int_0^1 \frac{1}{x^p} \, dx =$
(5) Solve each of the following separable ODEs:

(a) \( y' + xy = 2x \).

(b) \( xy' = \tan(y) \).

(c) \( y' = \frac{1 - y^2}{2} \) where \( y \) is a function of \( t \).
(6) A tank initially contains 30 lbs of salt dissolved in 1,000 gallons of water at time $t = 0$. Brine that contains 0.2 lbs of salt per gallon is pumped into the tank at 5 gallons per minute and the well-mixed solution is drained at the same rate.

(a) Determine a function $y(t)$ for the amount of salt in the tank (in lbs) at time $t \geq 0$.

Hint: $y'(t) = \text{rate (lbs/min) of salt coming} - \text{rate (lbs/min) of salt going out.}$

$$y(t) =$$

(b) Determine the **equilibrium solution**, that is, $\lim_{t \to \infty} y(t)$.

$$\lim_{t \to \infty} y(t) =$$