Math 254 - Practice Problems (THIS IS NOT A SAMPLE TEST).

1. Let \( f(r, \theta) = \frac{1}{2} r^2 \theta \), \( r = \sqrt{x^2 + y^2} \) and \( \theta = \tan^{-1} \frac{y}{x} \). Draw a tree diagram and use the chain rule to find \( f_x \) and \( f_y \). Give answer in terms of \( x \) and \( y \) only.

2. Find the maximum value of \( f(x, y, z) = x^2yz \) on the sphere \( x^2 + y^2 + z^2 = 4 \).

3. Find an equation for the tangent plane to the ellipsoid \( x^2 + 2y^2 + 5z^2 = 99 \) at \((1, 3, 4)\).

4. Evaluate the double integral:
\[
\int_{-1}^{1} \int_{0}^{1-y^2} \sqrt{1-x} \, dxdy.
\]

5. Evaluate the double integral:
\[
\int_{0}^{2} \int_{0}^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dydx.
\]

6. Evaluate the triple integral:
\[
\int_{0}^{2} \int_{\sqrt{3-x^2}}^{\sqrt{3-y^2}} \int_{0}^{\sqrt{3-y^2}} \frac{1}{\sqrt{x^2+y^2}} \, dzdydx.
\]

7. Calculate the volume bounded by the planes \( x = 0 \), \( y = 0 \), \( z = 0 \), and \( x + 2y + 4z = 8 \).

8. Evaluate the double integral:
\[
\int \int_{D} (x^2 - y^2)^{2/3} \, dA,
\]
where \( D \) is given by the inequality \(|x| + |y| \leq 1\).

9. Find the distance from the origin to the plane containing the point \((0, -1, 5)\) and the line \( \mathbf{r}(t) = \langle 2 - t, -1 + 2t, 3 - t \rangle \).

10. Find the area of the triangle with vertices at \((1, 5, 3), (3, 4, 5)\) and \((8, 0, 9)\).

11. Let \( \mathbf{a}(t) = \langle 2 \sin t + t \cos t, 2 \cos t - t \sin t \rangle \), \( 0 \leq t \leq 2\pi \) be the acceleration of a particle in the \( xy \)-plane. If the particle starts at the origin with an initial velocity vector \( < -1, 0 > \), then find functions for velocity and position of the particle.
12. Find the linear approximation of \( f(x, y, z) = z \tan^{-1} xy \) at \((1, 1, 2)\). Use it to approximate \((1.99) \tan^{-1} ((.98)(1.03))\) to the nearest thousandth.

13. Classify the quadratic surface: \( z^2 + 5y^2 = x^2 + 4x + 3 \).

Be careful! You should first complete the square...

14. Find the limit of the function \( f(x, y) = \frac{7x^3 y}{x^6 + 4y^2} \) as \((x, y)\) approaches the origin or justify that it does not exist.

15. Calculate maximum rate of increase for the function \( f(x, y, z) = e^{x - \cos(\pi y z)} \) at \((0, -1, \frac{1}{2})\) and the direction in which it occurs.

16. Evaluate the double integral:
\[
\int_0^2 \int_{\sqrt{4-x^2}}^{\sqrt{4-y^2}} x \ dy \ dx.
\]

17. Evaluate the triple integral:
\[
\int \int \int_E x^2 + y^2 \ dV,
\]
where \( E \) is given by \(0 \leq z \) and \( x^2 + y^2 \leq z^2 \leq 4 - x^2 - y^2\).

18. Find an equation of the plane containing the points \((0, 1, -3), (-1, 0, -5), \) and \((1, 2, -4)\).

19. Let \( \ell \) be the line through the point \((0, 1, 2)\) in the direction of the point \((2, 1, 3)\). Find the intersection of \( \ell \) and the plane \( 5x - 2y - z = 7 \).

20. Determine a function for the curvature of \( \mathbf{r}(t) = \langle \frac{2}{3} t^3, t^2, 1 - t \rangle \). Use it to calculate the curvature at \( \langle \frac{16}{3}, 4, -1 \rangle \).

21. Find an integral expression for the length of the curve \( \mathbf{r}(t) = \langle 4t^2, \cos 3t, 2 + \sin 3t \rangle \) for \(0 \leq t \leq 4\).

22. Evaluate the double integral:
\[
\int_0^4 \int_1^5 \frac{e^{x/y}}{y^3} \ dy \ dx.
\]
23. Evaluate the limit or justify that it does not exist:

\[
\lim_{{(x,y) \to (0,0)}} \frac{x^5 \sin(y^2)}{4x^5y^2 + y^2}.
\]

24. Let \( z = u \sin v - u^2 v, \ u = \sqrt{y} + x^y \) and \( v = \sin^{-1}(xy) \). Find \( z_x \) and \( z_y \). Express in terms of \( x, y \) and simplify.

25. Find the maximum and minimum values of \( f(x, y) = x^4 + y^4 - 4xy + 7 \) on \([0, 3] \times [0, 2] \).

26. Find points on the surface \( xy^2z^3 = 2 \) that are closest to the origin.

27. Evaluate the double integral:

\[
\int_0^1 \int_{\sqrt{x}}^1 x \sin^{-1} y^5 \ dy \ dx.
\]

28. Find the area of the elliptical disk \((2x + 5y - 3)^2 + (3x - 7y + 8)^2 \leq 1\).

29. Evaluate the triple integral:

\[
\int \int \int_E xyz \ dV,
\]

where \( E \) is bounded above by \( x^2 + y^2 + z^2 = 8 \) and below by \( z^2 = x^2 + y^2 \).

30. Evaluate the double integral:

\[
\int_{-2}^{2} \int_{0}^{2-|x|} xy^2 \sqrt{1 + x^2y^2} \ dy \ dx.
\]

31. Let \( E \) be the solid given by \( x^2 + y^2 \leq 9 \) and \( 0 \leq z \leq 5 \). Suppose the solid has the density function \( \rho(x, y, z) = 6 - z \) in grams per cubic centimeter. Find the mass, and the \( z \)-coordinate of the center of mass. Using a symmetry argument, determine the coordinates of the center of mass.

32. Find the directional derivative of \( f(x, y) = 1 + xy - x^2 + y^2 \) at \((1, 2)\) in the direction given by the vector \( < -3, 4 > \).

33. Pick random textbook problems from the sections we have covered.