Math 254 - Practice Problems (THIS IS NOT A SAMPLE TEST).

1. Find the distance between \((1, 2, 3, 4)\) and \((1, -1, 0, 5)\).

2. Find the dot product of \(\langle 12, 7 \rangle\) and \(\langle -3, 1 \rangle\). Find an expression for angle between these vectors in terms of \(\cos^{-1}\).

3. Find the cross product of \(\langle 17, 10, 1 \rangle\) and \(\langle -2, 0, 1 \rangle\).

4. A 3000 lb car drives up a steep street with an angle of elevation of 30 degrees. Find the normal and tangential components of the force of gravity on the car. Hint: project the force vector onto a vector that points in the direction of the incline.

5. A plane travels at 500 mph in a direction 45\(^\circ\) east of north. A wind blows at 20 mph in a direction 60\(^\circ\) east of south. Find the true speed and true course (how many degrees east/west of north/south) of the plane relative to the ground.

6. Find the vector form for equation of a line segment between \((1, -7)\) and \((11, 3)\).

7. Find a vector form for the equation of the line through the points \((5, 2, 2, -3)\) and \((1, 0, -6, 2)\).

8. Do the lines \(\langle 1 + t, 3 - 5t, 4t \rangle\) and \(\langle 4s, -11 - s, 3 + 9s \rangle\) intersect? If they do, find the point(s) their intersection. If not, justify.

9. Find the \textit{arc length parametrization} of the curve \(\mathbf{r}(t) = \langle \sin t, 2 \sin t, \sqrt{5} \cos t \rangle\).

Hint: Express \(\mathbf{r}(t)\) in terms of \(s\) where 
\[
s(t) = \int_0^t |\mathbf{r}'(u)| \, du.
\]

10. Find the volume of the parallelepiped spanned by the vectors \(\langle 1, 2, 3 \rangle, \langle 2, 1, 0 \rangle, \langle 0, 4, -1 \rangle\).

11. Find the distance from the point \((1, 1, 1)\) to the line \(\mathbf{r}(t) = \langle 0, \sqrt{2}, -5 \rangle + t\langle -1, 0, 2 \rangle\).

12. Find the tangent line to the path \(\mathbf{r}(t) = \langle t \cos t, t \sin t, t \rangle\) at \(t = \frac{\pi}{3}\).
13. Classify the quadratic surface: \(x^2 + 5y^2 = z^2 + 1\).

14. Find the limit of the function

\[
f(x, y) = \frac{7x^3y}{x^6 + 4y^2}
\]

as \((x, y)\) approaches the origin or justify that it does not exist.

15. Find the interior angles of the triangle with vertices \((2, 3, 1), (3, 7, 1), (-2, 4, 1)\).

16. Find the distance from the \((1, 1, 1)\) to the plane that is perpendicular to the line \(\mathbf{r}(t) = \langle 7 + t, -6t, 7 - t \rangle\) and contains \((11, 1, -3)\).

17. Evaluate the integral:

\[
\int_0^1 \left( \cos(\pi t)i - 4te^{-t^2}j + \frac{1}{t^2 + 2t + 2}k \right) dt.
\]

18. Find an equation of the plane containing the points \((-7, 0, -2), (2, 5, 1), \) and \((-9, 11, 0)\).

19. Consider the curve of intersection between the parabolic cylinder \(x^2 = 2y\) and the surface \(3z = xy\). Find an integral expression for the arc length along this curve from the origin to \((6, 18, 36)\).

20. For the curve \(\mathbf{r}(t) = \langle t, -t, t^2 \rangle\), find the curvature. Also find an integral expression for the arc length from \((0, 0, 0)\) to \((5, -5, 25)\).

21. A particle starts at the origin with an initial velocity of \(\langle 1, -2, 4 \rangle\). Its acceleration function is \(\mathbf{a}(t) = 3t^2i - 12tj + k\). Find functions for velocity, speed and position.

22. Pick random textbook problems from the sections we have covered.