(1) (5 pts) Find the solution to the second order linear constant coefficient non-homogeneous IVP $y'' - 4y' - 5y = e^{2x}$, $y(0) = 0$, $y'(0) = 0$. Justify your answer. An answer without proper support is worth zero credit.

The general solution to $y'' - 4y' - 5y = 0$ is $y = C_1 e^{-x} + C_2 e^{5x}$ since the characteristic polynomial is $r^2 - 4r - 5 = (r+1)(r-5)$.

Suppose $y_p = Ae^{2x}$ is a particular solution to $y'' - 4y' - 5y = e^{2x}$.

Then $4Ae^{2x} - 4(2Ae^{2x}) - 5(Ae^{2x}) = e^{2x}$

$\rightarrow 4A - 8A - 5A = 1 \rightarrow 9A = 1 \rightarrow A = -\frac{1}{9}$.

So the general solution to $y'' - 4y' - 5y = e^{2x}$ is $y = -\frac{1}{9}e^{2x} + C_1 e^{-x} + C_2 e^{5x}$.

$y(0) = 0 \rightarrow 0 = -\frac{1}{9} + C_1 + C_2 \quad \Rightarrow \quad 6C_2 = \frac{1}{3}, \quad C_2 = \frac{1}{18}$

$y'(0) = 0 \rightarrow 0 = -\frac{2}{9} - C_1 + 5C_2 \quad \Rightarrow \quad C_1 = \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$.

Then $C_1 = \frac{1}{9} - \frac{1}{18} = \frac{1}{18}$.

So $y = -\frac{e^{2x}}{9} + \frac{e^{-x} + e^{5x}}{18}$.
(2) (5 pts) Find the general solution to the second order linear constant coefficient non-homogeneous o.d.e. \( y'' + 4y = 3 \sin(2x) \). Justify your answer. An answer without proper support is worth zero credit.

The gen. soln. of \( y'' + 4y = 0 \) is \( y = c_1 \cos(2x) + c_2 \sin(2x) \).

Suppose \( y_p = x(A \cos(2x) + B \sin(2x)) \) is a particular soln. to \( y'' + 4y = 3 \sin(2x) \).

\[
y_p' = A \cos(2x) + B \sin(2x) + x(-2A \sin(2x) + 2B \cos(2x))
\]

\[
y_p'' = -4A \sin(2x) + 4B \cos(2x) + x(-4A \cos(2x) - 4B \sin(2x))
\]

\[
y_p'' + 4y_p = -4A \sin(2x) + 4B \cos(2x) = 3 \sin(2x)
\]

So \( A = -\frac{3}{4} \) & \( B = 0 \).

So the gen. soln. is

\[
y = -\frac{3}{4} x \cos(2x) + c_1 \cos(2x) + c_2 \sin(2x)
\]