(1) Let $A \in M_{mn}$. Show that $\ker A = \ker A^*A$.

Hint: $\|Ax\|^2 = (Ax, Ax) = (x, A^*Ax)$.

(2) Orthogonally diagonalize each (symmetric) matrix:

(a)

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$ 

(b)

$$B = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}.$$ 

(3) Show that the product of unitary matrices is unitary.

(4) Show that $f : \mathbb{C} \to \mathbb{C}$ given by $f(a + bi) = b + ai$ is a rigid motion, but not a linear transformation (regarding $\mathbb{C}$ as a one-dimensional complex vector space).

(5) Recall that $A \in M_{nn}$ is unitarily equivalent to $B$ if there is a unitary $U \in M_{nn}$ such that $A = UBU^*$. Produce a unitary $U$ showing that the matrix below is unitarily equivalent to a diagonal matrix $D$:

$$A = \begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix}.$$ 

Hint: Diagonalize it using an orthonormal basis!