(1) Let $n \geq 2$ be an integer. For each set of matrices, determine if it is a subgroup of the general linear group, $GL_n = \{A \in M_n|\det(A) \neq 0\}$ (invertible $n \times n$ matrices). Justify your answers.

(a) Diagonal $n \times n$ matrices with no 0s on the main diagonal.
(b) Invertible $n \times n$ matrices with no 0s on the main diagonal.
(c) All $n \times n$ matrices with determinant 2.
(d) All $n \times n$ matrices whose determinant is positive.

(2) Let $n \geq 2$ be an integer and $i, j \in \{1, 2, ..., n\}$ be distinct elements. Let $\sigma \in S_n$ (recall that $S_n$ the symmetric group on $\{1, 2, ..., n\}$). Show that $\sigma^{-1}(i \ j)\sigma = ((i)\sigma(j)\sigma)$ (using the convention that elements of $S_n$ act on $\{1, 2, ..., n\}$ on the right).

(3) Let $G$ be a group. Show that if $H \leq G$ and $K \leq G$ then $H \cap K \leq G$.

(4) Find the number of elements in the indicated cyclic group:

(a) The cyclic subgroup of $\mathbb{Z}_{60}$ generated by 55.
(b) The cyclic subgroup of $U = \{z \in \mathbb{C} : |z| = 1\}$ generated by $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

(5) Let $n \in \mathbb{N}$. Let $G = S_n$.

(a) Show that $G$ is non-abelian for $n \geq 3$.
(b) Show by example that every proper subgroup of a non-abelian group may be abelian.