1. Evaluate the following summations:

(a) \( \sum_{k=1}^{10} k \)

(b) \( \sum_{m=1}^{6} \)

(c) \( \sum_{n=1}^{4} n^2 \)

(d) \( \sum_{i=1}^{5} (1 + i^2) \)

2. Write the following in summation notation.

(a) \( 1 + 3 + 5 + \cdots + 99 \)

(b) \( 4 + 9 + 14 + \cdots + 44 \)

(c) \( 3 + 8 + 13 + \cdots + 63 \)

(d) \( \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50} \)
3. Let $f$ be the function graphed below.

(a) Give an **underestimate** for the shaded area in Figure A by adding up the areas of all the squares that lie **entirely inside** the region bounded by the graph of $f$, the $x$-axis and on the interval $[2, 6]$. (See Figure B).

(b) Give an **underestimate** for the shaded area in Figure A by adding up the areas of all the squares that **cover** the region bounded by the graph of $f$, the $x$-axis and on the interval $[2, 6]$. (See Figure C).

(c) Now average the overestimate and underestimates above.

(d) Is this a better estimate for the area of the region in Figure A than the two estimates from above?

(e) Will this always be the case? If not, sketch the graph of a function where the average is not as good of an estimate for the area as at least one of the under and overestimates.
4. Consider the interval $I = [a, b]$. A **partition of $I$ into $n$ subintervals** is a collection of subintervals, $[x_0, x_1], [x_1, x_2], [x_2, x_3], \ldots, [x_{n-1}, x_n]$, where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. Furthermore, the partition is **regular** if all the subintervals have equal width.

(a) Find the width of the interval $I$.

(b) Find the width, $\Delta x$, of a subinterval in a regular partition of $I$ into $n$-subintervals.

5. For each partition of $I$ into $n$ subintervals, we use $x^*_i$ to denote a point chosen from $[x_{i-1}, x_i]$ (a so called "sample point"). For instance, with a regular partition, $x^*_i = a + i\Delta x$ for $i = 1, 2, \ldots, n$ would run through successive right endpoints.

(a) What would run through left endpoints? Give a formula like the one above.

(b) What would run through the midpoints?
6. Given a regular partition of $I$ into $n$ subintervals and sample points $x^*_i$, the sum of "signed rectangular areas" given by $\sum_{i=1}^{n} f(x^*_i) \Delta x$ is called the **Riemann Sum** of $f$ over $I$.

(a) Let $f(x) = x^2 - 2x$. Write down and evaluate a Riemann sum over a regular partition of the interval $[1, 5]$ into 8 subintervals using right endpoints as sample points.

(b) Graph $y = x^2 - 2x$ over the interval $[1, 5]$ and draw the corresponding rectangles used for each subinterval of the Riemann sum you evaluated above.

7. A rock falling on the Moon has an acceleration of $a(t) = -1.6$ meters per second. Suppose an astronaut throws a rock into the air.

(a) Find *all* functions that differentiate to $a(t)$.

(b) What is the connection between the functions that differentiate to $a(t)$ and the velocity of the thrown rock?

(c) Suppose it was thrown with up at a speed of 10 meters per second. Use the previous parts to find the velocity $v(t)$ of the thrown rock at time $t$.

(d) When is the rock at its highest point?