Problem 1. What are the antiderivatives of \( \frac{1}{x^2 + 16} \)?

(a) \( 4 \arctan(x) + C \)

(b) \( \arctan\left(\frac{x}{4}\right) + C \)

(c) \( \frac{1}{4} \arctan\left(\frac{x}{4}\right) + C \)

(d) \( \arctan(4x) + C \)

(e) None of the above

Problem 2. Consider a function \( f(x) \) whose values are given in the table below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td>-7</td>
</tr>
</tbody>
</table>

Which of the following is an approximation for \( \int_0^{10} f(x) \, dx \) using right Riemann sums with 5 equal-sized subintervals (\( n = 5 \))?

(a) \( -8 \)

(b) \( 8 \)

(c) \( -12 \)

(d) \( 12 \)

(e) None of the above.
Problem 3. Which differentiation rule gives rise to substitution rule in integration?

(a) Chain Rule
(b) Power Rule
(c) Fundamental Theorem of Calculus
(d) Product Rule
(e) None of the above

Problem 4. What substitution should we make to find the following integral?

\[ \int t^3 \cos(t^4) \, dt \]

(a) \( u = t^3 \)
(b) \( u = t^4 \)
(c) \( u = \cos t \)
(d) \( u = \cos(t^4) \)
(e) None of the above
**Problem 5.** Which definite integral is equal to \( \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{1 + \left(2 + \frac{8k}{n}\right)^2} \cdot \frac{8}{n} \)?

(a) \( \int_{0}^{2} \frac{1}{1 + x^2} \, dx \)

(b) \( \int_{2}^{10} \frac{1}{1 + x^2} \, dx \)

(c) \( \int_{2}^{10} \frac{1}{2 + x^2} \, dx \)

(d) \( \int_{2}^{8} \frac{1}{2 + x^2} \, dx \)

(e) None of the above

**Problem 6.** Suppose that \( f(3) = -7, \ f(9) = 7 \) and \( H(x) = \int_{x^2}^{1} f(t) \, dt \). Evaluate \( H'(3) \).

(a) 42

(b) -42

(c) 21

(d) -21

(e) 1
Problem 7. Which one of the following is the average value of function \( f(x) = \cos(2x) \) on \([0, \frac{\pi}{8}]\)?

(a) \( \frac{2\sqrt{2}}{\pi} \)

(b) \( \frac{2}{\pi} \)

(c) 0

(d) \( 2\pi^2 \)

(e) None of the above.

Problem 8. The population of a community of foxes was 20 foxes when population measurements began \((t = 0)\); \(t\) is measured in years. The growth rate in units of foxes/year was observed to be

\[ P'(t) = 4 + 8 \cos \frac{\pi t}{4} \]

The population 32 years later is

(a) 146

(b) 152

(c) 130

(d) 102

(e) None of the above
Problem 9. \( \int \frac{4x^2 - 5x}{\sqrt{x}} \, dx = \)

(a) \( x^\frac{5}{2} - x^\frac{3}{2} + C \)
(b) \( \frac{8}{5} x^\frac{5}{2} - \frac{10}{3} x^\frac{3}{2} + C \)
(c) \( \frac{10}{3} x - \frac{8}{5} x^\frac{5}{2} + C \)
(d) \( \frac{8}{5} x^\frac{3}{2} - \frac{10}{3} x + C \)
(e) \( \frac{1}{5} x - \frac{10}{3} x^\frac{3}{2} + C \)

Problem 10. Consider \( \int_{1}^{5} (1 + x^2) \, dx \).

Which of the following is the midpoint Riemann sum for this definite integral using \( n \) subintervals?

(a) \( \sum_{k=1}^{n} \left( 1 + (1 - (k - 1) \frac{4}{n})^2 \right) \frac{4}{n} \)
(b) \( \sum_{k=1}^{n} \left( 1 + (1 + k \frac{4}{n})^2 \right) \frac{4}{n} \)
(c) \( \sum_{k=1}^{n} \left( 1 + (1 + (k - 1) \frac{4}{n})^2 \right) \frac{4}{n} \)
(d) \( \sum_{k=1}^{n} \left( 1 + (1 + (k - \frac{1}{2}) \frac{4}{n})^2 \right) \frac{4}{n} \)
(e) None of the above.
Problem 11. Consider

\[ \int_{0}^{4} (x^2 - 2x) \, dx. \]

Write down and evaluate the left Riemann sum for this definite integral using 4 equal-sized subintervals.
Problem 12. Suppose the vertical velocity, in feet per second, of a toy rocket launched straight up into the air is approximately given by \( v(t) = 96 - 32t \) at \( 0 \leq t \leq 10 \) seconds after launch.

1. (6 pts) Use the velocity function above to find the vertical displacement of the rocket for \( 0 \leq t \leq 4 \). Report your answer with the correct units. You may leave the answer unsimplified (e.g. \( 11(9^3) - 9^2 \)).

2. (6 pts) Use the velocity function above to find the vertical displacement of the rocket for \( 2 \leq t \leq 6 \). Report your answer with the correct units. You may leave the answer unsimplified.

3. (6 pts) Use the velocity function above to find the distance traveled by the rocket for \( 2 \leq t \leq 6 \). Report your answer with the correct units. You may leave the answer unsimplified.
Problem 13. Use a change of variables to find the following indefinite integrals

1. $\int (x^2 + x)^{10}(2x + 1)dx$

2. $\int \frac{dx}{1 + 4x^2}$

3. $\int \frac{x}{\sqrt{x - 4}}dx$

4. $\int \frac{y^2}{(y + 1)^3}dy$

5. $\int \frac{\sin x}{\cos^2 x}dx$

6. $\int_0^3 \frac{x}{\sqrt{x + 1}} dx$

Hint: Change the limits of integration according to your substitution.
**Problem 1.** Set up the integral of the volume yielded by revolving the region bounded by $y = \sqrt{x - 1}$, $x = 0$, $y = 2$ and $y = 0$, about the $x$-axis using the **Shell Method**.

(a) $\int_{0}^{1} \pi [(2)^2 - 0] \, dx + \int_{1}^{5} \pi [(2)^2 - (x - 1)] \, dx.$

(b) $\int_{0}^{2} 2\pi y \left[(\sqrt{y} - 1) - (0)\right] \, dy.$

(c) $\int_{0}^{2} 2\pi y \left[(y^2 + 1) - (0)\right] \, dy.$

(d) $\int_{0}^{5} 2\pi y \left[(y^2 + 1) - (0)\right] \, dy.$

(e) $\int_{0}^{5} 7dy.$

**Problem 2.** Hooke’s Law states that the force required to stretch/compress a spring is proportional to the amount of stretch/compression from equilibrium. It takes a 200 N force to stretch and hold the spring 0.1 m from its equilibrium position. How much work is required to compress the spring 0.1 m from its equilibrium position?

(a) 20 N  
(b) 10 J  
(c) 40 J  
(d) 2000 J  
(e) 7 J
Problem 3. Which integral below computes the arc length of the curve $x = y^2$ from $(0,0)$ to $(4,2)$?

(a) $\int_{0}^{2} \sqrt{1 + \frac{1}{2x}} \, dx$

(b) $\int_{0}^{2} \sqrt{1 + 4y^2} \, dy$

(c) $\int_{0}^{4} \sqrt{1 + 4y^2} \, dx$

(d) $\int_{0}^{2} \sqrt{1 + \frac{1}{4x}} \, dx$

(e) $\int_{0}^{2} 7 \, dx$

Problem 4. A cylindrical water tank has a height of 4 meters and a radius of 2 meter. How much work is required to empty the tank from the top if it is full? ($\rho$ is the density of water and $g$ is the acceleration due to gravity)

(a) $8\pi \rho g$

(b) $\pi \rho g$

(c) $12\pi \rho g$

(d) $32\pi \rho g$

(e) $7\pi \rho g$
Problem 5. Suppose a thin rod that occupies the interval \([0, 3]\) has a linear density function that is \(\rho(x) = \sqrt{x} + 1\) in kg/m. Which of the following is the mass of the rod?

(a) \(\frac{17}{3}\) kg  
(b) \(\frac{14}{3}\) kg  
(c) \(\frac{11}{3}\) kg  
(d) \(\frac{4}{3}\) kg  
(e) \(\frac{1}{3}\) kg

Problem 6. \(\int 2^{x-1} \, dx =\) 

(a) \(\frac{2^x}{\ln(x)} + C\)  
(b) \(\frac{2^x}{2 \ln(2)} + C\)  
(c) \(\frac{2^x}{2} + C\)  
(d) \((x - 1)2^{x-2} + C\)  
(e) \(7 + C\)
Problem 7. Which statement about the Shell Method is NOT true?

(a) The Shell Method produces a definite integral.

(b) The Shell Method is a method to find the volume of solids generated by revolving a region about a line.

(c) When using the Shell Method for a solid generated by revolving a region about the $y$-axis the variable of the integral is $x$.

(d) The Shell Method is a slicing method.

(e) When using the Shell Method for a solid generated by revolving a region about the line $y = 2$ the variable of the integral is $y$.

Problem 8. Let $R$ be the region bounded by the curves: $x = y^2$ and $x = 4$. Which of the following integrals gives you the area of $R$?

(a) $\int_{0}^{2} 4 - y^2 \, dy$

(b) $\int_{-2}^{2} 4 - \sqrt{x} \, dx$

(c) $\int_{0}^{4} 2\sqrt{x} \, dx$

(d) $\int_{0}^{4} 4 - \sqrt{x} \, dx$

(e) $\int_{0}^{4} 7 \, dx$
Problem 9 Let $R$ be the region bounded by $y = \sin x$ and the $x$-axis for $x$ in $[0, \pi]$

(a) Set up, BUT DO NOT INTEGRATE, the integral using the **Disk Method** that gives the volume of the solid generated by revolving $R$ about the $x$-axis.

(b) Set up, BUT DO NOT INTEGRATE, the integral using the **Washer Method** that gives the volume of the solid generated by revolving $R$ about the line $y = 2$.

(c) Set up, BUT DO NOT INTEGRATE, the integral using the **Shell Method** that gives the volume of the solid generated by revolving $R$ about the $y$-axis.

(d) Set up, BUT DO NOT INTEGRATE, the integral using the **Shell Method** that gives the volume of the solid generated by revolving $R$ about the line $x = -1$. 
Problem 10 Consider the following curve $9y^2 = x^3$ from $(0, 0)$ to $(10, \frac{10\sqrt{10}}{3})$.

(a) Set up the integral with respect to $x$ that gives the length of the curve on the given interval.

(b) Evaluate the integral.
Problem 11 A thin bar of length 2 meters is made of 2 different materials and has the following density function (the unit of the density is kg/m)

\[ \rho(x) = \begin{cases} 
 x^3 & 0 \leq x \leq 1 \\
 x^2(2-x) & 1 < x \leq 2 
\end{cases} \]

Find the mass of the bar.
Problem 12 Find the volume of the solid whose base is a semicircle of radius 1 and whose cross-sections perpendicular to this base, and perpendicular to the diameter of the semicircle, are squares.
Problem 13 Determine whether the following equality is true or not:

\[ \int_{0}^{1} x - x^2 \, dx = \int_{0}^{1} \sqrt{y} - y \, dy. \]

Can you justify your answer without evaluating either integral!?
Problem 14 Below is the graph of \( f(x) \) on the interval \([-4, 4]\). It consists of two line segments and a semicircle.

Evaluate \( S_a = \int_{-a}^{a} \sqrt{1 + (f'(x))^2} \, dx \) for each value of \( a \) below:

(a)  \( a = 0 \).

(b)  \( a = 2 \).

(c)  \( a = 4 \)
Problem 15 Below is the graph of $f(x)$ and $g(x)$ on the interval $[0, 1]$. The graphs intersect at $(0, 0)$ and $(1, 2)$.

Using the Shell Method, setup an integral that computes the volume of the solid generated by revolving the region bounded by these two curves about (a) the $y$-axis, (b) the $x$-axis and (c) the line $x = 1$.

Hint: Your answers should be in terms of $f(x)$, $g(x)$, $f^{-1}(y)$ and $g^{-1}(y)$.