1) Compute $F'(x)$ when:

   a) $g(x) \ F(x) = \int_{\frac{x}{2}}^{x} \cos(t^2)dt$
   
   b) $F(x) = \int_{\frac{x}{2}}^{x} \cos(t^2)dt$
   
   c) $F(x) = \int_{2x}^{x} \cos(t^2)dt$

2) The figure below shows the graph of a function $f(x)$ that has a continuous third derivative. The dashed lines are tangent to the graph of $y = f(x)$ at the points $(0, 2)$ and $(4, 1)$.

Based on what is shown, determine, if possible, whether the following integrals are positive, negative, or zero. (Hint! This lab has a title.)

   (a) $\int_{0}^{4} f(x) \ dx$
   
   (b) $\int_{0}^{4} f'(x) \ dx$
   
   (c) $\int_{0}^{4} f''(x) \ dx$
   
   (d) $\int_{0}^{4} f'''(x) \ dx$
3) The function $f'(x)$ is graphed below and we define $g(x) = \int_0^x f'(t)\,dt$ with $g(0) = 0$.

Assume the graph does not extend past what is visible. Use interval notation $(a, b)$ where appropriate. Provide a mathematical explanation in each part of the problem using information from the given graph.

a) Estimate $g(3)$.

b) List all intervals where $g(x)$ is negative.

c) List all intervals where $g(x)$ is increasing.

d) List all intervals where $g(x)$ is concave down.

e) Give all values of $x$ for which $g(x)$ has an extreme value and classify it as a local min/max.

f) Sketch the graph of $g(x)$ on the interval $[0, 4]$. Please draw the graph of $g(x)$ so it has the correct concavity.
4) Recall: A function $f(x)$ is even if $f(-x) = f(x)$ for all $x$ in the domain of $f$.
A function $g(x)$ is odd if $-g(-x) = g(x)$ for all $x$ in the domain of $g$.

Explain completely why the following statements are true.

a) If $f(x)$ is an even function, then
   \[ \int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx. \]

b) If $g(x)$ is an odd function, then
   \[ \int_{-a}^{a} g(x) \, dx = 0. \]

5) Suppose you know that $f(x)$ is an even function.

a) Now let \( \int_{-2}^{2} f(x) \, dx = \sqrt{3} \) and \( \int_{-2}^{2} f(x) \, dx = 10 \). What is \( \int_{-5}^{5} f(x) \, dx \)?

b) Now let \( \int_{-2}^{2} f(x) \, dx = \sqrt{3} \) and \( \int_{-2}^{2} f(x) \, dx = 10 \). What is \( \int_{0}^{5} f(x) \, dx \)?
6) A baseball is hit into the outfield on a parabolic trajectory where the height of the ball, in feet, is given by 
\[ y = 0.01(t + 1)(200 - t), \]
for any time \( t \), in seconds, between 0 and 200. Find the average height of the 
baseball over the horizontal extent of its flight. (Be sure to include units.)

7) Suppose \( f \) is a differentiable function. Several values of \( f(x) \) are given in the table below.

\[ \begin{array}{c|c|c|c|c|c}
   x & 0 & 3 & 5 & 10 \\
   f(x) & -1 & 3 & 4 & -9 \\
\end{array} \]

(a) Determine the average value of \( f'(x) \) on the interval \([0; 10] \).

(b) Suppose \( f \) is an even function. Determine the average value of \( f'(x) \) on the interval \([-5, 5] \).