Math 254 Recitation Activity - Fall 2019 - Week 10

Instructions: Please work in groups for these problems. You should write your solutions on a separate piece of paper when appropriate.

(1) If a function is odd with respect to one variable (e.g. \( f(x, -y, z) = -f(x, y, z) \)) and the region of the integral is symmetric about the plane (or line) where that variable is zero then the integral of the function over the region is zero. Explain how this lets us know that \( \iiint_E xz^2 \, dV \) where \( E \) is the solid region inside the ellipsoid \( x^2 + y^2 + 4z^2 = 36 \) and above the plane \( z = 2 \) is equal to 0 without evaluating.

(2) Evaluate \( \iiint_E z \, dV \) where \( E \) is...
   (a) ... the region between the circular cones \( z = \sqrt{x^2 + y^2} \) and \( z = 2 - \sqrt{x^2 + y^2} \).
   (b) ... the region between the circular cones \( z = \sqrt{x^2 + y^2} \) and \( z = \sqrt{3x^2 + 3y^2} \) that lies below the plane \( z = \sqrt{3} \).
   (c) ... the region above the circular cone \( z\sqrt{3} = \sqrt{x^2 + y^2} \) and inside the ellipsoid \( x^2 + y^2 + 3z^2 = 8 \).

(3) Evaluate \( \iiint_E \frac{1}{(x^2 + y^2 + z^2)^2} \, dV \) where \( E \) is the region above the plane \( z = \sqrt{3} \) and below the sphere \( x^2 + y^2 + z^2 = 4 \).

(4) Evaluate \( \iiint_E \frac{1}{x^2 + y^2} \, dV \) where \( E \) is the region inside the sphere \( x^2 + y^2 + z^2 = 4 \) outside the cylinder \( x^2 + y^2 = 3 \).

(5) Evaluate \( \iiint_E (x + y)^2 \, dV \) where \( E = \{ (x, y, z) | 1 \leq x^2 + y^2 + z^2 \leq 9, z \geq 0 \} \).

(6) Evaluate \( \int_0^1 \int_{\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} e^{\sqrt{x^2+y^2+z^2}} \, dz \, dx \, dy \)

(7) A ball of radius 1 ft occupies the region inside the unit sphere \( x^2 + y^2 + z^2 = 1 \) and has density at \( (x, y, z) \) given by \( f(x, y, z) = 2 - \sqrt{x^2 + y^2 + z^2} \) in lbs per cubic ft. Find the weight of the ball in lbs.
(8) Write down at least 2 of the 5 other ways to express the integral
\[ \int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x, y, z) \, dy \, dz \, dx. \]