Math 254 Recitation Activity - Fall 2019 - Week 4

Instructions: Please work in groups for these problems. You should write your solutions on a separate piece of paper when appropriate.

(1) Let $z = f(x, y) = \sqrt{25 - x^2 - 4y^2}$.

   (a) Draw the domain of $f(x, y)$ as a shaded region in the $xy$-plane.

   (b) Draw and label the level curves (or point) for $z = k$ corresponding to the values $k = 0, 2, 4, 5$.

(2) Find the level curves for $z = f(x, y)$ where $z$ is determined by the plane $ax + by + cz = d$, where $a \neq 0$, $b \neq 0$ and $c \neq 0$. Explain why it is that the level curves form parallel lines in the $xy$-plane.

(3) Determine the domain and draw at least 3 level surfaces of $f(x, y, z) = \sqrt{10 - x^2 - y^2 - z^2}$.

(4) Let $f(x, y) = \frac{2x^3y}{x^6 + y^2}$ and $m$ be a scalar.

   (a) Consider the line $x = 0$. Determine $\lim_{(0, y) \to (0, 0)} f(0, y)$.

   (b) Consider the line $y = mx$. Determine $\lim_{(x, mx) \to (0, 0)} f(x, mx)$.

   (c) What conclusion does the above let you make about $\lim_{(x, y) \to (0, 0)} f(x, y)$?

   (d) Consider the cubic curve $y = x^3$. Determine $\lim_{(x, x^3) \to (0, 0)} f(x, x^3)$.

   (e) What conclusion does the above let you make about $\lim_{(x, y) \to (0, 0)} f(x, y)$?

(5) Now consider $g(x, y) = \frac{2x^2y}{x^2 + y^2}$.

   (a) Let’s use polar coordinates: Let $x = r \cos (\theta)$ and $y = r \sin (\theta)$ to express $g$ as a function of $r$ and $\theta$, that is $g(r, \theta)$.

   (b) Now use the squeeze theorem to justify that $\lim_{(x, y) \to (0, 0)} \frac{2x^2y}{x^2 + y^2} = 0$. 


(6) Suppose that $z = f(x, y)$ is a smooth surface and that below are some level curves of $z = f(x, y)$. Estimate the slope of an oriented path along the surface at the points $P$ and $Q$ where (a) the path is in the positive $x$-direction and (b) the path is in the positive $y$-direction.