Math 254 Recitation Activity - Fall 2019 - Week 6

Instructions: Please work in groups for these problems. You should write your solutions on a separate piece of paper when appropriate.

(1) Below is a topographical map. The curves shown are the level curves of the elevation function \( E(x, y) \) (altitude at a point \( x \) meters east and \( y \) meters north of the origin (“Start”).

(a) Estimate \( E_x(200, 175) \) and \( E_y(200, 175) \).

(b) Suppose you are walking in the park. In which direction from \((200, 175)\) should you go so that your elevation increases most rapidly? Express your answer as a vector.

(c) Draw vectors with their tails at points \( P_1(600, 600) \) and \( P_2(900, 1100) \) to represent \( \nabla E(P_1) \) and \( \nabla E(P_2) \). (Scale is arbitrary, but draw the vectors such that their relative lengths are reflected.) Hint: you do not need to make any numerical estimates.

(2) If a surface in \( xyz \)-space is given by \( F(x, y, z) = k \) for some differentiable function \( F(x, y, z) \) and \( P \) is a point on this surface then \( \nabla F(P) \) is orthogonal to the surface at \( P \) and can be used to find an equation for the tangent plane \( ax + by + cz = d \) to the surface at \( P \). For each surface below, find an equation for the tangent plane at \( P \).

(a) The surface \( 4x^2 + 9y^2 - z^2 = 0 \) at the point \((2, 1, -5)\).

(b) The surface \( z = x^2 + y^2 \) at the point \((1, 1, 2)\).

(c) The graph of \( f(x, y) = \frac{x}{x + y} \) at \((4, -2, 2)\).
(3) Let \( h(x, y) = \sqrt{x^4 + y^2} \). Use a linear approximation to approximate \( h(x, y) \) when \((x, y)\) changes from \((2, 3)\) to \((2.1, 2.9)\).

(4) Suppose that below are level curves of \( g(x, y) \). Estimate the following:

(a) The directional derivative of \( g \) at the point \((1, 1)\) in the direction \( \mathbf{v} = \langle 1, 0 \rangle \).

(b) The directional derivative of \( g \) at the point \((-1, -1)\) in the direction \( \mathbf{v} = \langle 2, 1 \rangle \).

(c) \( \nabla g(2, 0.5) \).