Math 254 Recitation Activity - Fall 2019 - Week 7

Instructions: Please work in groups for these problems. You should write your solutions on a separate piece of paper when appropriate.

A function \( f(x, y) \) has a global maximum (minimum) at \((x_0, y_0)\) if \( f(x_0, y_0) \) is the largest (smallest) value in the range of \( f(x, y) \).

(1) If \( f(x, y) \) is defined on a bounded region \( R \), that contains its boundary (called a “closed” region) and \( f(x, y) \) is continuous on \( R \) then \( f(x, y) \) has a global maximum and minimum on \( R \). To find them, compare the value of \( f(x, y) \) at critical points, that are points within the boundary of \( R \) (called “interior points”), with any potential extreme points on the boundary.

EXAMPLE: Consider \( f(x, y) = e^{4x+y^2-x^2} \) on the region \( x^2 + y^2 \leq 9 \).

\[
  f_x = 2(2 - x)e^{4x+y^2-x^2} = 0 \text{ implies } x = 2. \quad f_y = 2ye^{4x+y^2-x^2} = 0 \text{ implies } y = 0.
\]

The point \((2, 0)\) lies within the interior (not on the boundary). So we keep it.

Now consider restricting the function to \( x^2 + y^2 = 9 \), or \( y^2 = 9 - x^2 \) with \(-3 \leq x \leq 3\):

Set \( g(x) = f(x, \pm \sqrt{9 - x^2}) = e^{4x+9-2x^2} \). So \( g'(x) = 2(1 - x)e^{4x+9-2x^2} = 0 \) at the point \( x = 1 \) (with \( y = \pm 2\sqrt{2} \)).

With the endpoints at \( x = \pm 3 \) and the interior point found above we get the following potential extreme points: \((-3, 0), (1, 2\sqrt{2}), (1, -2\sqrt{2}), (2, 0), (3, 0)\).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(f(x, y))</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3, 0))</td>
<td>(e^{-21})</td>
<td>Global minimum</td>
</tr>
<tr>
<td>((1, 2\sqrt{2}))</td>
<td>(e^{11})</td>
<td>Global maximum</td>
</tr>
<tr>
<td>((1, -2\sqrt{2}))</td>
<td>(e^{11})</td>
<td>Global maximum</td>
</tr>
<tr>
<td>((2, 0))</td>
<td>(e^4)</td>
<td>Not a global extreme</td>
</tr>
<tr>
<td>((3, 0))</td>
<td>(e^3)</td>
<td>Not a global extreme</td>
</tr>
</tbody>
</table>

Find the global extremes of ...

(a) ... \( f(x, y) = \ln(2 + y^2 + x^2 - 2x) \) on the region given by \( 4x^2 + y^2 \leq 16 \).

(b) ... \( f(x, y) = x^2\sqrt{x^2 + y^2 + 1} \) on the region given by \( x^2 - 1 \leq y \leq 3 \).

(c) ... \( f(x, y) = \frac{x^2+2xy+y^2}{x^2+y^2+1} \) on the region in the first quadrant given by \( 1 \leq x+y \leq 2 \).
(2) Find the minimum value of \( f(x, y, z) = 3x^2 + 2x - 2xy + 3y^2 - 2y - z \) on the paraboloid \( z = x^2 + y^2 - 1 \).

(3) Find the maximum value of \( f(x, y, z) = xyz^2 \) on the ellipsoid \( 9x^2 + 4y^2 + z^2 = 36 \).