Show all work clearly and justify your answers using clear logic and complete sentences. You have 35 minutes to take this 10 point quiz. Good luck!

1. (3 points) Determine whether the function

\[ f(n) = n + 1 \]

is one-to-one, onto, or both. Prove your answer. The domain and codomain of \( f \) are the set of all integers.

\[ f \text{ is one-to-one, since } \forall \, x \in \mathbb{Z}, \quad f(x-1) = x - 1 + 1 = x \quad \text{and} \quad f(x) = f(y) \quad \text{then} \quad x = y. \]

2. (4 points) By experimenting with small values of positive integers \( n \), guess a formula for the given sum,

\[ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}; \]

then use induction to verify your formula.

\[ \begin{align*}
\text{Claim: } & \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1} \\
\text{Base: } & \text{ Done.} \\
\text{Ind.: } & \text{ Let } \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1} \quad \text{ for } \quad n > 1, \, n \in \mathbb{Z}. \\
\text{Then } & \text{ we } + \frac{1}{n(n+1)} = \frac{n+1}{n+2}. \\
\end{align*} \]

\[ \Rightarrow \frac{1}{1 \cdot 2} + \cdots + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} \cdot \frac{n+2}{n+2} = \frac{n(n+2)+1}{(n+2)(n+1)} \]

(Continued on the back)

so result holds for \( n+1 \), hence \( \forall \, n \in \mathbb{Z} \).
3. (3 points) Let $X, Y$ and $Z$ be sets. Let $g$ be a function from $X$ to $Y$, and let $f$ be a function from $Y$ to $Z$. Prove or disprove (with a specific counter example) the following statement:

If $f$ is onto, then $f \circ g$ is onto.

[Hint: drawing an arrow diagram is useful!]

Let $g = \{(1, a), (2, a), (3, a)\}$ and

$$f = \{(a, d), (b, d), (c, e)\}.$$ Then

$$f \circ g = \{(1, d), (2, d), (3, d)\}$$ is not onto,

since $\exists x \in X$ s.t. $f \circ g(x) = e$. 