1. **(2 points)** Expand \((2c - 3d)^5\) using the Binomial Theorem. Do not simplify.

**Proof.** By the Binomial Theorem, we have

\[
(2c - 3d)^5 = C(5, 0)(2c)^5 + C(5, 1)(2c)^4(-3d) + C(5, 2)(2c)^3(-3d)^2 \\
+ C(5, 3)(2c)^2(-3d)^3 + C(5, 4)(2c)(-3d)^4 + C(5, 5)(-3d)^5.
\]

2. **(2 points)** Draw \(K_3\) and \(K_5\). (These are the connected graphs on 3 and 5 vertices, respectively.)

**Proof.** \(K_3\) is the connected graph with 3 vertices, so just draw the connected graph with each vertex of degree 2. Similarly for with 5 vertices.

3. **(3 points)** Use the Binomial Theorem to show that

\[
0 = \sum_{k=0}^{n} (-1)^k C(n, k).
\]

[Hint: \(0=1+(-1)\)].

**Proof.** We are assuming \(n \geq 1\) here. Since \(0 = 1 - 1 = (1 - 1)^n\) for \(n \geq 1\), we have, by the Binomial Theorem,

\[
0 = (1 - 1)^n = \sum_{k=0}^{n} C(n, k)1^{n-k}(-1)^k = \sum_{k=0}^{n} C(n, k)(-1)^k.
\]
4. (3 points) Find a formula for the number of edges in $K_n$. You must explain your reasoning to receive full credit!

Proof. We can count the number of edges by counting the incident edges at each of the $n$ vertices. So first, at one vertex, there will be $n - 1$ edges incident on this vertex, as this is a vertex in $K_n$. Now at the next vertex, there will be also $n - 1$ incident edges, but one will have already been counted by the previous vertex. Hence this next vertex will have $n - 2$ unique edges to count. Continue in this process for all the rest of the vertices, adding up the number of edges. You will get that the number of edges in $K_n$ is

$$(n - 1) + (n - 2) + \cdots + 2 + 1 = \sum_{i=1}^{n-1} i = \frac{n(n - 1)}{2}.$$ 

For a different (shorter) proof, note that we can count the edges by choosing two vertices from the set of $n$ vertices (the order of the vertices we pick won’t matter, this will avoid overcounting of edges). That is, the answer is also $C(n, 2) = \frac{n(n-1)}{2}$. $\square$