Extra Final Exam Review Exercises:

IMPORTANT: These items relate to just the material post the second exam (sections 7.3-7.5 and 7.7-7.9). Since the final is cumulative, you should also go over both your midterms, all the recitation activities, and the other review materials posted.

(1) Evaluate $\int \cos^3(x) \, dx$.

(2) Evaluate $\int \sec^3(x) \tan^5(x) \, dx$.

(3) Evaluate $\int \cos^2(x) \sin^2(x) \, dx$.

(4) Evaluate $\int \frac{1}{x^4\sqrt{1 - 4x^2}} \, dx$.

(5) Evaluate $\int \frac{x - 1}{x^3 + x} \, dx$.

(6) Which first order differential equation is NOT separable?
   
   (a) $y' = y$
   (b) $y' = 1 - y^2$
   (c) $y' = \frac{y}{t} + y$
   (d) $y' = \frac{y}{t} + t$
   (e) $y' = \frac{y^2}{1 + t^2}$

(7) Determine the Trapezoid Rule estimate for $\int_0^6 \frac{x}{1 + x^2} \, dx$ using 6 subintervals.

(8) Determine the Midpoint Rule estimate for $\int x^2 \, dx$ on $[0, 8]$ using 4 subintervals.

(9) Determine the Simpson’s Rule estimate for $f(x) = \int_0^4 0.5x^4 - x \, dx$ using 8 subintervals.

(10) Evaluate the integral $\int \frac{1}{e^{2x} - e^x} \, dx$.
    
    Hint: Begin with the substitution $u = e^x$.

(11) Evaluate the integral $\int \frac{\sqrt{x^2 - 9}}{x} \, dx$ assuming $x > 3$.

(12) Evaluate $\int_0^2 \frac{x^3}{\sqrt{4 + x^2}} \, dx$. 

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(13) Determine the value of improper integral \( \int_{2}^{\infty} \frac{1}{(x + 2)^2} \, dx \) if convergent or state that it diverges.

(14) Determine the value of the improper integral \( \int_{1}^{\infty} \frac{x^3 + x - 1}{x^4 + x^2} \, dx \) if convergent or state that it diverges.

(15) Determine the value of improper integral \( \int_{0}^{1} \frac{e^x}{\sqrt{e^x - 1}} \, dx \) if convergent or state that it diverges.

(16) Find \( y(1) \) given that \( y' = 2y \) and \( y(0) = 5 \).

(17) Find the general solution to \( y' = \frac{y^2 + 1}{e^{2x}} \).

(18) Find the general solution to \( y' = xe^{x-y} \).

(19) Find \( y \) such that \( y' = y^2 \ln \left( \frac{1}{t} \right) \) and \( y(1) = 1 \).

Hint: \( \ln (x^r) = r \ln (x) \).