Math 254 Written HW 1 - Fall 2019

**Instructions:** Put your solutions on a separate piece(s) of paper. You may use this page as a cover sheet. **Submission for credit:** Submit it at the START of recitation on Thursday, October 3.

Generally it is good to explain your work: an important facet of mathematics is explaining how you arrived at the answer, and not just stating the answer. (On exams you will be required to show your work.)

These problems include some review material that is relevant to the entire course.

(1) Assume that $1 = 2\cos(\theta)$. Find the smallest positive value for $\theta$ that makes the equation true. This should be an exact answer and not a decimal approximation. The angles that are important for you to know are $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and the angles that have these as reference angles.

(2) Let $f(t) = 2t^2$, $g(t) = \frac{8\sqrt{2}}{3}t^{3/2}$, $h(t) = 4t$. Compute

$$\int_1^3 \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2} \, dt.$$ 

Check your work as you go so that errors do not propagate. Look for ways to use algebra to simplify the expression in the intermediate steps. Show your work.

(3) A rectangular block has dimensions $5 \text{ cm} \times 8 \text{ cm} \times 10 \text{ cm}$. What is the length of its longest diagonal? Include units.

(4) See the figure above. Express the vector $\overrightarrow{AB}$ using angular bracket notation and using the coordinate unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. (Handwritten: $\vec{i}$, $\vec{j}$, $\vec{k}$.)
(5) A submarine climbs at an angle of $\pi/6$ radians ($30^\circ$) above the horizontal with a heading to the northeast. If its speed is 20 knots, find the components of the velocity in the east, north, and vertical directions.

Use the standard convention that the positive $x$ direction is east, the positive $y$ direction is north, and the positive $z$ direction is up.

(6) Give a geometric description of the set of points $(x, y, z)$ in $xyz$-space that satisfy both $x^2 + y^2 + z^2 = 25$ and $z = 4$.

(7) Consider a line in $xy$-plane given by $ax + by = c$, where $a, b, c$ are scalars. Show that the line is perpendicular to $\vec{n} = \langle a, b \rangle$.

Hint: Let $P(x_0, y_0)$ and $Q(x_1, y_1)$ be two distinct points on the line. Show that $\vec{PQ}$ is orthogonal to $\vec{n}$.

(8) For the following vectors $\mathbf{u}$ and $\mathbf{v}$, express $\mathbf{u}$ as the sum $\mathbf{u} = \mathbf{p} + \mathbf{n}$ where $\mathbf{p}$ is parallel to $\mathbf{v}$ and $\mathbf{n}$ is orthogonal to $\mathbf{v}$:

(a) $\mathbf{u} = \langle 3, 4 \rangle$, $\mathbf{v} = \langle 1, 1 \rangle$

(b) $\mathbf{u} = \langle 2, 2, 7 \rangle$, $\mathbf{v} = \langle 1, 1, 2 \rangle$