Math 254 Written HW 4 - Fall 2019

Instructions: Put your solutions on a separate piece(s) of paper. You may use this page as a cover sheet. Submission for credit: Submit it in recitation on Thursday, Oct. 24.

Generally it is good to explain your work: an important facet of mathematics is explaining how you arrived at the answer, and not just stating the answer. (On exams you will be required to show your work.)

(1) A wolf pack heads into mountains, going up and down, but always heading eastward. Suppose the pack has a starting elevation above sea level of 1 km and after \( t \) hours, for \( 0 \leq t \leq 2 \), their elevation above sea level is \( \cos \left( \frac{\pi t}{2} \right) + \frac{\pi t}{2} \sin \left( \frac{\pi t}{2} \right) \) km and their eastward displacement (from their initial position) is \( \sin \left( \frac{\pi t}{2} \right) - \frac{\pi t}{2} \cos \left( \frac{\pi t}{2} \right) \) km.

(a) Determine a vector-valued function that gives the position of the wolf pack at \( 0 \leq t \leq 2 \) hours.

(b) Determine the distance traveled by the wolf pack after 2 hours.

(c) At what average speed did the wolf pack travel during those 2 hours?

(2) A vector-valued function \( \mathbf{r}(t) \) for \( a \leq t \leq b \) is said to be an “arc-length parametrization” of a smooth curve \( C \) if the graph of \( \mathbf{r}(t) \) for \( a \leq t \leq b \) traverses \( C \) once and \( |\mathbf{r}'(t)| = 1 \) for all \( a \leq t \leq b \). Given such a parametrization of a curve \( C \), the arc-length along \( C \) between \( \mathbf{r}(t_0) \) and \( \mathbf{r}(t_1) \) is simply \( |t_0 - t_1| \).

Determine if each curve below is an arc-length parametrization of its graph for the indicated values of \( t \).

(a) \( \mathbf{r}_1(t) = \langle 2 \cos \left( \frac{t}{2} \right), 2 \sin \left( \frac{t}{2} \right) \rangle \) for \( 0 \leq t \leq 4\pi \).

(b) \( \mathbf{r}_2(t) = \langle 4t, 3 \cos (t), 3 \sin (t) \rangle \) for any \( t \geq 0 \).

(c) \( \mathbf{r}_3(t) = \langle 4t/5, 3 \cos \left( \frac{t}{5} \right), 3 \sin \left( \frac{t}{5} \right) \rangle \) for any \( t \geq 0 \).

(d) \( \mathbf{r}_4(t) = \langle t, 2t^{3/2}, 9 - t \rangle \) for any \( 0 \leq t \leq 9 \).

(3) A plane \( P \) contains the point \( (1, 2, 3) \) and is perpendicular to the line \( \vec{l}(t) = \langle t - 1, 1 - t, 2t + 3 \rangle \). Find an equation for this plane.

(4) Find the distance between the sphere \( (x + 1)^2 + (y + 2)^2 + (z + 2)^2 = 2 \) and the plane \( 2x + 2y + z = 4 \).

Hint: First find the distance between the center of the sphere and the plane.