Math 343 - Intro. to Modern Algebra **Homework 2** **Due: Feb. 1st, 2019**

Note: A random subset of these problems will be graded for credit.

(1) Suppose G is a finite binary structure under an associative product, where there is both left and right cancelation. Prove G is a group.

(2) Suppose G is a group. Prove that no matter how you bracket the product of any number of group elements, the result is the same (e.g., $(ab)(cd) = a(b(cd))$ for all $a, b, c, d \in G$).

Hint: Use induction (on the number of factors).

(3) Give an example of a group G for which $H = \{a \in G | a^2 = e\}$ is **NOT** a subgroup.

(4) Suppose $H, K \leq G$ and that the sets $KH = \{kh | k \in K, h \in H\}$ and $HK = \{hk | h \in H, k \in K\}$ are equal. Prove $HK \leq G$.

(5) Give an example of a group G and subgroups H, K such that HK is **NOT** a subgroup.

(6) Let G be a group and $H \leq G$. For $x \in G$ we have $Hx = \{hx | h \in H\}$ (called a right-coset of H in G). Show that for all $a, b \in G$ either $Ha = Hb$ or $Ha \cap Hb = \emptyset$.

(7) For each set of matrices, determine if it is a group (under matrix multiplication – which is associative). Justify your answers.

(a) Invertible 2×2 matrices with real coefficients with 0 below the main diagonal.

(b) Invertible 3×3 matrices with rational coefficients such that the main diagonal is all 0s.

(c) Invertible 3×3 matrices with rational coefficients such that the main diagonal is all 1s.

(d) $n \times n$ matrices with complex coefficients and determinant equal to 1, where n is any positive integer.

(8) Let G be a group and $g \in G$ be a fixed element. The *centralizer* of g in G is $C_G(g) = \{a \in G | ag = ga\}$. Show that $C_G(g) \leq G$.

(9) Let G be a group. Prove that if $(ab)^2 = a^2b^2$ for all $a, b \in G$ then G is abelian.