Show all work clearly. If a question asks for an explanation, justify your answers using complete sentences. You have 40 minutes to take this 10 point quiz. Good luck!

1. (2 points) Determine $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for the following rational function. Then give the horizontal asymptote of $f$ (if any).

$$f(x) = \frac{6x^2 - 9x + 8}{3x^2 + 2}.$$ 

$$\lim_{x \to \infty} \frac{6x^2 - 9x + 8}{3x^2} = \lim_{x \to \infty} \left( \frac{6 - \frac{9}{x} + \frac{8}{x^2}}{3 + \frac{2}{x^2}} \right) = 2$$

$$\lim_{x \to -\infty} \frac{6x^2 - 9x + 8}{3x^2} = \lim_{x \to -\infty} \left( \frac{6 - \frac{9}{x} + \frac{8}{x^2}}{3 + \frac{2}{x^2}} \right) = 2$$

The horizontal asymptote of $f$ is $y = 2$.

2. (3 points) Determine whether the following function is continuous at $x = 1$. Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ \frac{1}{3} & \text{if } x = 1 \end{cases}$$

$$f(1) \text{ is defined, but }$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} (x + 1) = 2 \neq \frac{1}{3}$$

So $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} \neq f(1)$, so $f$ is not continuous at $x = 1$. 
3. (8 points) Let \( a \) be a real number and let

\[
g(x) = \begin{cases} 
  x^2 + x & \text{if } x < 1 \\
  a & \text{if } x = 1 \\
  3x + 5 & \text{if } x > 1
\end{cases}
\]

(a) Determine the value of \( a \) for which \( g \) is continuous from the left at \( 1 \). [Hint: what is \( \lim_{x \to 1^-} g(x) \)?]

(b) Determine the value of \( a \) for which \( g \) is continuous from the right at \( 1 \). [Hint: what is \( \lim_{x \to 1^+} g(x) \)?]

(c) Is there a value of \( a \) for which \( g \) is continuous at \( 1 \)? Explain.

\[
\begin{align*}
&\text{a) } \lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} x^2 + x = 2, \text{ so take } a = 2. \\
&\text{b) } \lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} 3x + 5 = 8, \text{ so take } a = 8. \\
&\text{c) No value exists, since } \lim_{x \to 1} g(x) \text{ DNE.}
\end{align*}
\]

4. (9 points) Let \( f(x) = 3x^2 + 2x - 10 \) and let \( a = 1 \).

(a) Find the derivative function \( f' \) for \( f \). Must use \( \lim_{h \to 0} \) def.!

(b) Find an equation of the line tangent to the graph at \( f \) at \( (a, f(a)) \) for the given value of \( a \).

(c) Graph \( f \) and the tangent line.

\[
\begin{align*}
&\text{a) } \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 + 2(x+h) - 10 - 3x^2 - 2x + 10}{h} \\
&\quad = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 10 - 3x^2 - 2x + 10}{h} \\
&\quad = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h} = \lim_{h \to 0} (6x + 3h + 2) = 6x + 2. \\
&\quad \text{So } f'(x) = 6x + 2. \\
&\text{b) If } a = 1, \text{ then } f(1) = -5, \text{ hence } f'(1) = 8 \text{ and by the point-slope formula, } y + 5 = 8(x - 1). \\
&\quad \text{Thus, } y = 8x - 13 \text{ is the equation.}
\end{align*}
\]