Representations and Sunspot Stability

George W. Evans
University of Oregon

Bruce McGough
Oregon State University

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Abstract

By endowing his agents with simple forecasting models, or *representations*, Woodford (1990) found that finite state Markov sunspot equilibria may be stable under learning. We show that common factor representations generalize to all sunspot equilibria the representations used by Woodford (1990). We find that if finite state Markov sunspots are stable under learning then *all* sunspots are stable under learning, provided common factor representations are used.

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1 A brief history of stable sunspots

Sunspot equilibria provide avenues through which agents' expectations can drive fluctuations in real economic activity. Interest in these equilibria developed through the work of Shell (1977), Azariadis (1981), Cass and Shell (1983) and Guesnerie (1986), but remained couched primarily in the theoretical literature until Benhabib and Farmer (1994) and Farmer and Guo (1994) demonstrated the existence of sunspot equilibria in RBC-type models modified to incorporate externalities or monopolistic competition: see Farmer (1999) for a detailed development. These authors, and many other since, have used calibrated DSGE models to argue that fluctuations in agents' expectations explain at least part of the business cycle. These arguments have been extended to New Keynesian monetary models: Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004) suggest that passive monetary policy in the seventies produced an economic environment conducive to sunspot equilibria and the associated high volatility.

The simple existence of sunspot equilibria in a model does not imply their relevance: it may not be possible for agents to coordinate their behavior appropriately. A benchmark coordination device in macroeconomics is stability under learning: for details see Evans and Honkapohja (2001).

In an OLG model, Woodford (1990) found that finite state Markov sunspots may be stable under learning, thus lending credence to the relevance of sunspots for applied models, and in part inspiring the work of Farmer and others. However, Evans and Honkapohja (2001) found that the sunspots studied by Farmer and Guo were not stable under learning. This finding was further supported by Evans and McGough (2005a) and Duffy and Xiao (2007) who searched for stable sunspots in a host of RBC-type models and found none.

The stability of equilibria may depend on the values of the model's parameters; this has been known since Bray and Savin (1986). However, that Woodford's sunspots are stable and Farmer's sunspots are not cannot be explained so easily; indeed, as is well known and shown below, in Woodford's model, Farmer's sunspots are *never* stable. Perhaps then the explanation lies in the stochastic properties of the equilibria in question. Woodford considers equilibria that follow a finite state Markov chain, and Farmer's equilibria are autoregressive with conditional noise captured by a martingale difference sequence with continuous support. However, as we will see below, the stochastic nature of an equilibrium has no impact on its stability.

The stability of sunspot equilibria turns on the way they are viewed by private agents. More formally, a given equilibrium may often be identified with a particular recursion: for example, Farmer considers equilibria in AR(1) form. We called these recursions *representations*. When investigating stability under learning, a representation specifies a natural functional form for the forecasting model agents estimate and use to form their expectations. In Evans and McGough (2005b), we showed for a model with sunspots that a given equilibrium may have several representations consistent with it, and further, that stability under learning is representation dependent. In particular, we found that sunspot equilibria previously thought to be unstable under learning become stable if agents use a forecasting model consistent with a *common factor representation*.

This notion of representation allows us to fully investigate why, in Woodford's model, Farmer's sunspots are never stable and Woodford's sunspots sometimes are. In this paper we show that the representation used in Woodford's analysis is, in fact, a special case of a common factor representation; indeed, common factor representations generalize to all sunspot equilibria the learning mechanism used by Woodford for finite state Markov processes. We conclude that whenever these finite state Markov sunspots are stable under learning, all sunspot equilibria will be, provided a common factor representation is used.

2 Woodford's model: a linearization

The non-stochastic linearized version of Woodford's model is sufficient for our purposes, and is given by

$$y_t = \beta E_t y_{t+1}, \tag{1}$$

where $y_t \in \mathbb{R}$. A rational expectations equilibrium (REE) is any bounded stochastic process y_t satisfying (1). We consider only doubly-infinite processes.

Let y_t be an REE, and set $\varepsilon_t = y_t - E_{t-1}y_t$. Then y_t satisfies the recursion

$$y_t = \beta^{-1} y_{t-1} + \varepsilon_t. \tag{2}$$

We call this recursion the *general form representation* of the equilibrium y_t. Because y_t is bounded we know that either $\varepsilon_t = 0$ (so that $y_{t+k} = 0$ for $k \in \mathbb{Z}$) or $|\beta| > 1$; in the latter case, ε_t can be any martingale difference sequence (mds) with finite support. For the remainder of the paper we assume $|\beta| > 1$. Note that y_t is an REE of (1) if and only if there exists an mds ε_t so that y_t satisfies (2). The mds ε_t captures variation in y_t due to fluctuations in agents' expectations; it is often called a sunspot and the associated REE y_t is often called a sunspot equilibrium.

To analyze stability under learning, private agents are given a forecasting model whose functional form is consistent with a representation of the REE; this forecasting model is sometimes called a *perceived law of motion* or PLM. For example, to study the stability of an REE y_t, agents might be given the PLM $y_t = \theta' X_t$, where $X_t = (1, y_{t-1}, \varepsilon_t)'$ is the vector of regressors.[1] Using this PLM, agents form expectations which are then imposed into (1) to determine the actual behavior of y_t. The equation identifying this behavior is called the *actual law of motion* or ALM, and under appropriate conditions, it takes the same functional form as the PLM: $y_t = T(\theta)' X_t$. Note that the map $T : \mathbb{R}^3 \to \mathbb{R}^3$ takes the perceived coefficients to the actual coefficients, and an REE may be identified with a fixed point θ^* of this map.

Using the T-map, we may write down the ordinary differential equation $\dot{\theta} = T(\theta) - \theta$; a rest point θ^* of this ode corresponds to an REE of (1). We say that the REE is *E-stable* if it is a (locally asymptotically) stable fixed point of the ode. Note that the stability of the fixed point obtains if the eigenvalues of the T-map's derivative, DT, have real part less than one.[2] We focus on E-stability of equilibria because of the *E-stability Principle*, which states that E-stable REE are locally learnable under

[1] This PLM includes a constant to capture the fact that the linearized model has been recentered at its deterministic steady state – a value not assumed known by agents. Also, normally the PLM would include a perceived error; including this error would be distracting and would not alter our results.

[2] In case the stability of sunspot equilibria in linear models is being analyzed, the appropriate E-stability condition is that the eigenvalues of DT be less than *or equal* to unity: for details see Evans and Honkapohja (2001).

least squares or related learning algorithms. The E-stability Principle is known to apply to many models, including (1). For details on the E-stability Principle and the deep connection between E-stability and stochastic convergence of statistical learning algorithms, see Evans and Honkapohja (2001).

3 Results

Let y_t be an REE of (1), and consider its stability by providing agents with a PLM consistent with the representation (2):

$$y_t = \theta' X_t = \theta_1 + \theta_2 y_{t-1} + \theta_3 \varepsilon_t.$$

The actual law of motion is

$$y_t = T(\theta)' X_t = \beta(1+\theta_2)\theta_1 + \beta\theta_2^2 y_{t-1} + \beta\theta_2\theta_3\varepsilon_t,$$

giving the T-map

$$\begin{array}{rcl} \theta_1 & \rightarrow & \beta(1+\theta_2)\theta_1 \\ \theta_2 & \rightarrow & \beta\theta_2^2 \\ \theta_3 & \rightarrow & \beta\theta_2\theta_3 \end{array} \tag{3}$$

Computing the Jacobian yields

$$DT = \begin{pmatrix} \beta(1+\theta_2) & \beta\theta_1 & 0 \\ 0 & 2\beta\theta_2 & 0 \\ 0 & \beta\theta_3 & \beta\theta_2 \end{pmatrix}. \tag{4}$$

Evaluating at the fixed point $\theta^* = (0, \beta^{-1}, \theta_3)$ yields an eigenvalue equal to two, so that the REE is unstable.[3] We conclude with the well-known result that y_t is not stable under learning, and since y_t was arbitrary, no sunspot equilibria of (1) are learnable – at least if agents use a PLM consistent with general form representations.

Equilibria generated by coordination on an arbitrary mds were the type studied by Farmer and others in applied models; however, Woodford had a different type of sunspot in mind. Take as primitive a 2-state Markov process $s_t \in \{0, 1\}$, with transition matrix π. For any $\bar{y} \in \mathbb{R}^2$, we may construct the associated Markov process

$$y_t = \bar{y}_i \Leftrightarrow s_t = i - 1, \text{ for } i = 1, 2. \tag{5}$$

Evans and Honkapohja (2003) showed that y_t is an REE of (1) if and only if the following two conditions hold:

$$\pi_{11} + \pi_{22} = 1 + \beta^{-1} \tag{6}$$

$$\bar{y}_2(1 - \pi_{11}) = -\bar{y}_1(1 - \pi_{22}). \tag{7}$$

[3] Here, θ_3 can be any real number, reflecting the fact that if ε_t is an mds then so too is $\theta_3\varepsilon_t$.

In case y_t is an REE, we call (5) its *natural representation*. These y_t are the 2-state Markov sunspots for the linear model (1) analogous to those studied in Woodford (1990).

To study the stability under learning of these types of equilibria we follow Evans and Honkapohja (2003): we specify a PLM consistent with (5); that is, we assume agents observe s_t, know that the transition matrix for s_t is π, and believe the equilibrium of the model is a 2-state Markov process; but we assume they do not know the values of y_t in these two states.[4] Instead, we provide agents with perceived values for the states, and thus we may identify their perceptions with points $\tilde{y} \in \mathbb{R}^2$. For given perceptions $\tilde{y}$, the PLM is formally given by

$$y_t = \tilde{y}_i \Leftrightarrow s_t = i - 1, \text{ for } i = 1, 2. \tag{8}$$

To obtain the ALM, notice that if agents observe $s_t = 0$ then they will forecast y_{t+1} as

$$\hat{E}_t(y_{t+1}|s_t = 0) = \pi_{11}\tilde{y}_1 + (1 - \pi_{11})\tilde{y}_2.$$

It follows that if $s_t = 0$ then

$$y_t = \beta\left(\pi_{11}\tilde{y}_1 + (1 - \pi_{11})\tilde{y}_2\right).$$

A symmetric computation holds for $s_t = 1$, so that if agents have perceptions given by $\tilde{y}$ then the economy follow a 2-state Markov process with transition matrix π and states $\hat{y} \in \mathbb{R}^2$ given by

$$\hat{y} = \beta\pi\tilde{y} \equiv T_N(\tilde{y}). \tag{9}$$

We call $T_N : \mathbb{R}^2 \to \mathbb{R}^2$ the "natural T-map."

When perceptions and truth coincide, that is, when $T_N(\tilde{y}) = \tilde{y}$, an REE is identified. It follows that $\tilde{y}$ is an REE if and only if the matrix $\beta\pi$ has a unit eigenvalue and $\tilde{y}$ is an associated eigenvector: precisely these conditions are guaranteed by the restrictions (6) and (7).

Now notice that $DT_N = \beta\pi$. Since the eigenvalues of DT_N evaluated at a fixed point are unity and β, we obtain Evans and Honkapohja's conclusion that a 2-state Markov sunspot equilibrium y_t is stable under learning provided $\beta < -1$. This is the result for the linear model analogous to the celebrated stability result of Woodford (1990).

Above we noted that if y_t is an REE then there is an mds ε_t so that y_t satisfies (2); thus finite state Markov sunspot equilibria must also have general form representations. Also, even in this case, if agents use a forecasting model consistent with (2)

[4]Incorporating an error into the PLM, as discussed above, makes realistic the assumption that agents perceive the economy is a 2-state process even though the observations do not support this perception.

then the equilibrium is unstable. We conclude, as did Evans and Honkapohja, that stability under learning of finite state Markov sunspots is representation dependent.

We now turn to the main question of this paper: "What's so special about finite state Markov sunspots?" The answer, of course, is "Nothing." We will now show that if $\beta < -1$ then all sunspots are stable under learning, provided agents have perceptions consistent with Woodford's natural PLM. To facilitate the argument, we first show how to write Woodford's finite state PLM in a way that naturally generalizes to all sunspot equilibria, regardless of support cardinality; and to do this, we begin by constructing the general form representation of a 2-state Markov sunspot equilibrium $\bar{y} \in \mathbb{R}^2$ (the equilibrium is again associated to the fundamental process s_t with transition matrix π).

Define a stochastic process $\varepsilon_t(s_{t-1}, s_t)$ as follows:

s_{t-1}	s_t	$\varepsilon_t(s_{t-1}, s_t)$
0	0	$(1-\beta^{-1})\bar{y}_1$
0	1	$\bar{y}_2 - \beta^{-1}\bar{y}_1$
1	0	$\bar{y}_1 - \beta^{-1}\bar{y}_2$
1	1	$(1-\beta^{-1})\bar{y}_2$

Because the 2-state process $(\bar{y}, \pi)$ satisfies the restrictions (6) and (7) it can be shown that ε_t is a martingale difference sequence, and further, by construction, the 2-state process $(\bar{y}, \pi)$ solves

$$y_t = \beta^{-1} y_{t-1} + \varepsilon_t. \tag{10}$$

Equation (10) is the general form representation of the 2-state sunspot equilibrium $\bar{y}$.

Using the lag operator, we may solve (10) for y_t to obtain

$$\begin{aligned} y_t &= \eta_t \\ \eta_t &= (1-\beta^{-1}L)^{-1}\varepsilon_t. \end{aligned} \tag{11}$$

Equation (11) is the *common factor representation* of the 2-state sunspot equilibrium $\bar{y}$. Because y_t is a 2-state Markov process, it follows that η_t is a 2-state Markov process. We think of η_t as a serially correlated extrinsic noise process on which agents coordinate to form expectations, and we call it a *common factor sunspot*. For an extended discussion of common factor representations and their relation to minimal state variable solutions, see Evans and McGough (2005b).[5]

[5]The name "common factor representation" comes from their construction, which may be thought of as obtained by dividing out the common factor $(1-\beta^{-1}L)$. This construction is less trivial in higher dimensions and with the incorporation of lags into the reduced form model.

To study the stability under learning of y_t, we provide agents with a PLM consistent with (11):

$$y_t = a + b\eta_t. \tag{12}$$

Notice that because η_t is a 2-state Markov process, the perceptions identified by this PLM are entirely analogous to the perceptions identified by Woodford's natural PLM (8): agents believe the economy follows a 2-state Markov process. We compute

$$E_t y_{t+1} = a + bE_t\eta_{t+1},$$

and because

$$E_t\eta_{t+1} = \beta^{-1}\eta_t$$

we find that the T-map associated to a common factor representation is

$$T_{CF}(a, b) = (\beta a, b).$$

It follows that the eigenvalues of DT_{CF} are one and β. We conclude that if agents use common factor representations to form their forecasting models then y_t is stable under learning provided $\beta < -1$, just like Woodford.

Our stability argument was made in the context of an mds constructed to replicate the 2-state Markov sunspot $(\bar{y}, \pi)$; however, *nothing* in the argument relied on the 2-state nature of the equilibrium: indeed, the common factor representation was intentionally constructed to be independent of the specific properties of the mds ε_t. To see this, let y_t be *any* REE of the model, and let ε_t be the associated martingale difference sequence. Let $\eta_t = (1 - \beta^{-1}L)^{-1}\varepsilon_t$. Then $y_t = \eta_t$. Again, provide agents with the PLM (12). Exactly the same T-map obtains. We conclude that if $\beta < -1$ then *all* sunspot equilibria are stable under learning, provided agents use a common factor representation as their forecasting model.

While this result generalizes Woodford's stability result to all sunspots, we have yet to formally establish the connection between the stability of common factor representations and the stability of natural representations. Recall our primitive assumption that agents believe the economy follows a 2-state Markov process with transition matrix π, where π satisfies (6). Let Γ be the set of all 2-state Markov processes with transition matrix π and notice that Γ may be identified with $\mathbb{R}^2$. We think of Γ as the set of all possible agent beliefs.

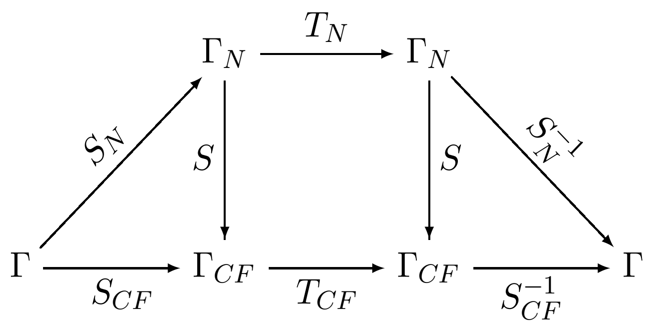
We may also think of Γ as the set of all PLMs consistent with natural representations: simply recall that s_t is the primitive sunspot with transition matrix π, let $y \in \Gamma$ and see equation (5). While the identification of Γ with this set is, in a sense, trivial, distinguishing the sets will aid clarity; therefore, let Γ_N be the set of all PLMs consistent with natural representations (as captured by $\mathbb{R}^2$ together with equation (5)) and let $S_N : \Gamma \to \Gamma_N$ be the identity map on $\mathbb{R}^2$. The map S_N, then, takes an agent's beliefs to the associated natural PLM.

Finally, we must characterize the set of all PLMs consistent with common factor representations. To this end, let y_t be a 2-state Markov sunspot equilibrium with transition matrix π and states $\bar{y} \in \mathbb{R}^2$ satisfying (7), let ε_t be the mds such that y_t solves (2), and let η_t be the common factor sunspot generated by ε_t. Recall that η_t is a 2-state Markov process. Now let Γ_{CF} be the set of PLMs consistent with the common factor representation of y_t, and notice that a PLM is uniquely determined by its coefficients: $a + b\eta_t = c + d\eta_t \Leftrightarrow a = c$ and $b = d$. Recalling that Γ is just $\mathbb{R}^2$, we may define $S_{CF} : \Gamma \longrightarrow \Gamma_{CF}$ by

$$S_{CF}(y) = \left(\frac{y_1(\bar{y}_2 - \bar{y}_1) - \bar{y}_1(y_2 - y_1)}{\bar{y}_2 - \bar{y}_1}, \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)'.$$

Straightforward computation shows that S_{CF} is a bijection. Finally, define $S : \Gamma_N \longrightarrow \Gamma_{CF}$ by $S = S_{CF} \circ S_N^{-1}$ and note that S is bijective by construction.

Proposition 1 *The following diagram commutes:*



A commutative diagram is an efficient way to make statements about equivalence of functions. To understand the diagram's meaning, start with a point in any set (the "initial" set) and pick any other set (the "final" set) that can be reached from the initial set by following a path of arrows. Under the composition of the functions corresponding to the arrows in your path, the point you chose in the initial set is mapped to a point in the final set. Because the diagram commutes, the path of arrows you chose to reach the final set is irrelevant. An important example is

$$S_N^{-1} \circ T_N \circ S_N = S_{CF}^{-1} \circ T_{CF} \circ S_{CF}. \tag{13}$$

To prove this statement, note that S_N is the identity map; so it suffices to show that for any point $y \in \mathbb{R}^2$ it follows that

$$S_{CF} \circ T_N(y) = T_{CF} \circ S_{CF}(y).$$

Notice that

$$\hat{y} = T_N(y) = (\beta(\pi_{11}y_1 + (1-\pi_{11})y_2), \beta((1-\pi_{22})y_1 + \pi_{22}y_2))'$$

so that, using (6), $\hat{y}_2 - \hat{y}_1 = y_2 - y_1$. Using this, direct computation yields

$$S_{CF} \circ T_N(y) = \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left(\beta(\pi_{11}y_1 + (1-\pi_{11})y_2)(\bar{y}_2 - \bar{y}_1) - \bar{y}_1(y_2 - y_1) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)'.$$

Using restrictions (6) and (7), we obtain that

$$\pi_{11}(\bar{y}_2 - \bar{y}_1) = \bar{y}_2 - \beta^{-1}\bar{y}_1.$$

Combining, we get

$$\begin{aligned}
S_{CF} \circ T_N(y) &= \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left(\beta\pi_{11}(\bar{y}_2 - \bar{y}_1)(y_1 - y_2) + \beta y_2(\bar{y}_2 - \bar{y}_1) - \bar{y}_1(y_2 - y_1) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)' \\
&= \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left((\beta\bar{y}_2 - \bar{y}_1)(y_1 - y_2) + \beta y_2(\bar{y}_2 - \bar{y}_1) - \bar{y}_1(y_2 - y_1) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)' \\
&= \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left(\beta(y_2(\bar{y}_2 - \bar{y}_1) - \bar{y}_2(y_2 - y_1)) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)' \\
&= \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left(\beta(-y_2\bar{y}_1 + \bar{y}_2 y_1) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)' \\
&= \left(\frac{1}{\bar{y}_2 - \bar{y}_1} \left(\beta(y_1(\bar{y}_2 - \bar{y}_1) - \bar{y}_1(y_2 - y_1)) \right), \frac{y_2 - y_1}{\bar{y}_2 - \bar{y}_1} \right)' \\
&= T_{CF} \circ S_{CF}(y).
\end{aligned}$$

The commutativity of the above diagram allows us to make several precise statements about the relationship between common factor representations and Woodford's natural representations.

Corollary 2 *PLMs consistent with common factor representations and PLMs consistent with natural representations identify the same set of agent beliefs.*

Corollary 3 *Viewed as acting on agents' beliefs, the maps T_N and T_{CF} coincide.*

Corollary 2 acknowledges that S is bijective and Corollary 3 is an interpretation of equation (13). Taken together, these corollaries indicate the sense in which, when restricted to finite state Markov sunspot equilibria, common factor representations may be identified with Woodford's natural representations; importantly, however, common factor representations may be used to analyze the stability of any sunspot equilibrium. In this sense, we may view common factor representations as a generalization of Woodford's natural representations to all sunspot equilibria.

Finally, by applying the chain rule and the inverse function theorem to equation (13) we find that the eigenvalues of DT_N and DT_{CF} coincide. Thus

Corollary 4 *Common factor representations and natural representations have the same stability properties.*

Furthermore, because the eigenvalues of the associated T-maps are independent of the cardinality of the sunspot's support, we may conclude that whenever finite state Markov sunspots are stable under learning, all sunspot equilibria are stable under learning, provided common factor representations are used for the stability analysis.[6]

4 Conclusion

The implications for learning stability of distinguishing between REE and their representations can be striking. For the forward-looking model, such as the linearized OG set-up used by Woodford, Evans and Honkapohja (2003) show that for a subset of the indeterminacy region, finite-state sunspot solutions can be stable under learning for a natural representation, while they are not stable when put into an AR(1) representation. One might think that this is a reflection of the type of sunspot equilibria being considered. In particular, one might hypothesize that in this model a sunspot solution with continuous support would never be stable under learning, since such a solution cannot be represented as a function of an exogenous finite-state Markov process. Our central finding is that this conjecture is incorrect. We show that for the subset of the indeterminacy region in which finite-state sunspot solutions are stable under learning using the natural representation, any sunspot equilibrium is also stable under learning, provided agents use what is known as the common factor representation of the sunspot solution.

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[6]While this conclusion only applies to the simple model considered in this paper, we conjecture that it holds more generally.

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