Chapter 4

Interest Rates and Rates of Return
Types of Bonds

- Simple loan
- Discount bond
- Coupon bond
- Fixed-payment loan

Figure 4.1
Time Lines for Credit Market Instruments
Present Value

- Comparing returns across debt types is difficult since timing of repayment differs.
- Solution is the concept of present value.
- The present value of $1 received $n$ years in the future is $\frac{1}{(1 + i)^n}$.

Compound Interest

- $100 in one year = $100(1 + i)$
- ($100 principle repayment +i($100) in interest)$
- EX: If i = 10%, in one year, $100 will be:$100(1.1) = $110 ($100 principle + $10 interest)
- If you reinvest $110 at 10%, then at the end of two years you will have $110(1.1) = $121
- OR $100(1.1)(1.1) = $100(1.1^2) = $121
Present Value

If $121 = 100(1.1)^2$, then $121$ is the future value (FV) of $100$ today. We found this by compounding interest.

Alternatively, we could divide both sides of the above equation by $(1.1)^2$:

$$\frac{121}{(1.1)^2} = 100$$

In this form we call the $100$ the present value (PV) of receiving $121$ in two years.

PRESENT VALUE

- In general, $PV = \frac{FV}{(1 + i)^n}$

Where FV is the future value
PV is the present value
i is the interest rate
n is the number of years in the future you have to wait before receiving the FV.
Bond yields and Prices

- The price of a debt instrument is equal to its Present Value

- EX: One period discount bond with a face value of $100:
  - If $i= 10\%$, $PV = \frac{100}{(1.1)} = 90.90$
  - If $i= 5\%$, $PV = \frac{100}{(1.05)} = 95.24$

**PRICE OF A COUPON BOND**

- $PV = \text{PRICE} = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$
- Current Yield $= \frac{C}{P}$
- Coupon Yield $= \frac{C}{FV}$
A CONSOL BOND

- A coupon bond with infinite maturity is a perpetual, or consol, bond.

- The price of a consol is:

  \[ PV = \frac{C}{i} \]

Present Value

- Comparing returns across debt types is difficult since timing of repayment differs.

- Solution is the concept of present value.

- The present value of $1 received \( n \) years in the future is $1/(1 + i)^n$. 
Which is the best deal?

- Signing bonus today: $3,000,000
- In 1 year: $6,000,000
- In 2 years: $7,000,000
- In 3 years: $8,000,000

- Signing bonus today: $6,000,000
- In 1 year: $5,500,000
- In 2 years: $6,000,000
- In 3 years: $6,000,000

Current Price and Face Value

- If current price = face value, then yield to maturity = current yield = coupon rate.
- If current price < face value, then yield to maturity > current yield > coupon rate.
- If current price > face value, then yield to maturity < current yield < coupon rate.
Total Rate of Return

- The total rate of return is the sum of current yield and actual capital gain or loss.
- Rate of return can differ from yield to maturity.
- The formula for total rate of return is:
  \[ R = \frac{C}{P_t} + \frac{(P_{t+1} - P_t)}{P_t}. \]

Real and Nominal Interest Rates

- Expected real interest rate = nominal interest rate - the expected rate of inflation.
- Fisher hypothesis: change in expected inflation = change in nominal interest rate.
- The real rate of return equals the nominal rate of return adjusted for expected inflation.
Figure 4.3 Real and Nominal Interest Rates, 1960-2000

Nominal rate increases, real rate decreases