Modeling decisions: influence diagrams and probabilistic networks
You are here

The

Identify the decision situation and objectives

Identify the management alternatives

Decompose and model the problem

Identify the best alternative

Perform sensitivity analysis

Is further analysis needed?

YES

NO

Implement the best alternative

From Clemen and Reilly 2001
Influence Diagram

- **The decision**: Improve habitat?

- **Uncertain events**: Future Population size, Current Population size

- **Consequence**: Conservation value (outcome, utility)
Influence Diagram

**Improve habitat?**

- **Future Population size**
- **Current Population size**

- Arrows represent causality
- Arrows represent flow of information (no feedbacks)

- **Conservation value**

NOT a flowchart
Influence Diagram

- Improve habitat?
  - Future Population size
    - Conservation value
      - Root node
  - Current Population size
Influence Diagram

**Improve habitat?**

**Future Population size**

**Current Population size**

No link: Current population size is not known with certainty when decision is made

ONLY type of link to represent timing and flow of information (current population size is known when decision is made)

**Conservation value**

No link: Current population size is not known with certainty when decision is made
Influence Diagram: imperfect information

- Improve habitat?
- Field Sampling results
- Current Population Size?
- Future Population size
- Conservation value

Field Sampling results

Current Population Size?

Future Population size

Conservation value

Improve habitat?
Influence Diagram: linked/sequential decisions

- Improve habitat?
- Sample population?
- Field sampling results
- Current Population Size?
- Future Population size
- Conservation value

Influence Diagram: linked/sequential decisions

- Improve habitat?
- Sample population?
- Field sampling results
- Current Population Size?
- Future Population size
- Conservation value
Influence Diagram: linked/sequential decisions

- Restoring habitat?
- Reintroduce species?
- Species persistence
- Future habitat
- Conservation value
Influence Diagram: Dynamic (Markovian) decisions

- Harvest Decision Time $t = 1$
- Harvest Decision Time $t = 2$
- Harvest Decision Time $t = 3$

Population size $t = 1$
Population size $t = 2$
Population size $t = 3$

Total Harvest $t=1$
Total Harvest $t=2$
Total Harvest $t=3$

Cumulative harvest
Common problems

Failing to account for direct effects of decisions on consequences

- Improve habitat?
  - Cost of action = free

- Future Population size

- Current Population size

- Conservation value
Common problems

Failing to account for direct effects of decisions on consequences

1. Improve habitat?
   - Cost of action NOT free
2. Future Population size
   - Current Population size
3. Conservation value

Current Population size

Future Population size

Cost of action NOT free

Improve habitat?

Conservation value
Common problems

Missing important uncertainties

Improve habitat?

Future habitat

Conservation value

Current Population size

Future Population size
Common problems

Failing to consider the current state of the system

Current habitat

Improve habitat?

Future habitat

Conservation value

Future Population size

Current Population size
Common problems

Miss-specifying the relationships

OR
Common problems

Value of B depends on A, $B = Ax$

Value of C depends of B, $C = Bx$

IF B known with certainty, the value of A no longer affects C

C is conditionally independent of A

*Is B really necessary?*
Common problems

Joint effects of A and B on C

\[ C = A + B \]

- \( B \) "high"
- \( B \) "low"

\[ C = A + B + A*B \]

- \( B \) "high"
- \( B \) "low"
Common problems

Joint effects of A and B on C

The value of B depends on A and the value of C depends on the value of A and B

Fairly rare, make sure this is what you meant
Interpret the relationships for timber harvest Influence Diagram

Timber harvest decision

Stream Habitat

Socioeconomic value

Current Population Size

Future Population Size
Interpret the relationships for timber harvest *Influence Diagram*
Defining Nodes

Potential States

- **Timber harvest decision**
  - No, Yes
  - Low, Medium, High
  - 0, 10ha, 100ha

- **Stream Habitat**
  - Good, Bad
  - Excellent, Good, Poor, Bad

- **Current Population Size**
  - Low, Medium, High
  - 0, 1-10, 10-20

States are mutually exclusive and collectively exhaustive
Defining Node States

States must be explicitly defined
Transparency avoids confusion
Values more explicit than narratives

States can be discretized continuous values

States often based on ecological/management considerations
Timber Harvest Influence Diagram

- **Timber harvest decision**: yes, no
- **Stream habitat**: good, poor
- **Current population size**: small, large
- **Future population size**: small, large
- **Socioeconomic value**

Uncertain events
Bayesian Belief Network
(DAG, probabilistic network, causal network)

Remove the decision and utility.....

Stream habitat
  good, poor

Current population size
  small, large

Future population size
  small, large

where future fish population size is influenced by stream habitat and current fish population size
**Conditional Probabilities**

- **Stream habitat**
  - good, poor

- **Current population size**
  - small, large

- **Future population size**
  - small, large

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th>Current population size</th>
<th>Future population size</th>
<th>Sums to 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>good</td>
<td>small</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>good</td>
<td>large</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>poor</td>
<td>small</td>
<td>0.6</td>
<td>0.4</td>
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<tr>
<td>poor</td>
<td>large</td>
<td>0.5</td>
<td>0.5</td>
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</table>
Unconditional Probabilities

The top two nodes do not depend on anything else in the model.

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>good</td>
<td>0.5</td>
</tr>
<tr>
<td>poor</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current population size</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>0.8</td>
</tr>
<tr>
<td>large</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Sum to 1
Decision Tree

Stream habitat

- good
  - small: 0.80
  - large: 0.20

- poor
  - small: 0.50
  - large: 0.50

Future population size

- small: 0.30
  - large: 0.70
- small: 0.10
  - large: 0.90
- small: 0.60
  - large: 0.40
- small: 0.50
  - large: 0.50
Estimating probability future population is small

Future population size

Stream habitat

Current population size

Calculation

\[0.50 \times 0.80 \times 0.30 + 0.50 \times 0.20 \times 0.10 + 0.50 \times 0.80 \times 0.60 + 0.50 \times 0.20 \times 0.50 = 0.42\]
## Bayesian Belief Network

### Stream habitat

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>good</td>
<td>0.5</td>
</tr>
<tr>
<td>poor</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Current population size

<table>
<thead>
<tr>
<th>Current population size</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>0.8</td>
</tr>
<tr>
<td>large</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Future population size

<table>
<thead>
<tr>
<th>Future population size</th>
<th>42.0</th>
<th>58.0</th>
</tr>
</thead>
</table>

### Probability Matrix

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th>Current population size</th>
<th>Future population size</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>Probability</td>
<td>State</td>
</tr>
<tr>
<td>good</td>
<td>0.5</td>
<td>small</td>
</tr>
<tr>
<td>poor</td>
<td>0.5</td>
<td>large</td>
</tr>
<tr>
<td>good</td>
<td>0.5</td>
<td>small</td>
</tr>
<tr>
<td>poor</td>
<td>0.5</td>
<td>large</td>
</tr>
</tbody>
</table>

**Column sum:**

- small: 0.42
- large: 0.58
Interpreting the graphical model in Netica

<table>
<thead>
<tr>
<th>Name of node</th>
<th>States</th>
<th>Stream habitat</th>
<th>Current population size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>good</td>
<td>50.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>poor</td>
<td>50.0</td>
</tr>
</tbody>
</table>

State-specific probabilities (expressed as %)

Bar length represents size of probability

Model created with Netica software
Bayesian Belief Networks, conditional independence

Stream habitat now depends on the width of the riparian zone

<table>
<thead>
<tr>
<th>Riparian width</th>
<th>50.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wide</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th>47.5</th>
<th>52.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>good</td>
<td></td>
<td></td>
</tr>
<tr>
<td>poor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current population size</th>
<th>80.0</th>
<th>20.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td></td>
<td></td>
</tr>
<tr>
<td>large</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Future population size</th>
<th>42.8</th>
<th>57.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td></td>
<td></td>
</tr>
<tr>
<td>large</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Bayesian Belief Networks, conditional independence

<table>
<thead>
<tr>
<th>Riparian width</th>
<th>Stream habitat</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow</td>
<td>good 20.0</td>
</tr>
<tr>
<td></td>
<td>poor 80.0</td>
</tr>
<tr>
<td>wide</td>
<td>good 80.0</td>
</tr>
<tr>
<td></td>
<td>poor 80.0</td>
</tr>
</tbody>
</table>

Current population size
- small: 80.0
- large: 20.0

Future population size
- small: 51.6
- large: 48.4

Predicted probabilities when riparian is narrow
Bayesian Belief Networks, conditional independence

<table>
<thead>
<tr>
<th>Riparian width</th>
<th>Stream habitat</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow</td>
<td>0</td>
</tr>
<tr>
<td>wide</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stream habitat</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>good</td>
<td>75.0</td>
</tr>
<tr>
<td>poor</td>
<td>25.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current population size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>80.0</td>
</tr>
<tr>
<td>large</td>
<td>20.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Future population size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>34.0</td>
</tr>
<tr>
<td>large</td>
<td>66.0</td>
</tr>
</tbody>
</table>

Predicted probabilities when riparian is wide

<table>
<thead>
<tr>
<th>Riparian</th>
<th>Stream habitat</th>
</tr>
</thead>
<tbody>
<tr>
<td>width</td>
<td>good</td>
</tr>
<tr>
<td>narrow</td>
<td>0.20</td>
</tr>
<tr>
<td>wide</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Bayesian Belief Networks, conditional independence

Assume that new habitat was good.

Riparian width is wide.

Predicted probabilities

Future population size
- small: 26.0
- large: 74.0

Current population size
- small: 80.0
- large: 20.0

Stream habitat
- good: 100
- poor: 0

Riparian width
- narrow: 0
- wide: 100
Bayesian Belief Networks, conditional independence

Riparian width is narrow

Assume that knew habitat was good

Predicted Probabilities do not change

Once the condition of Stream habitat is known Riparian width no longer affects Future population status.
Consequences of independence

Modularity

Physical habitat model

- Timber harvest
- Precipitation
- Streambed sediment
- Soil disturbance

Fish-habitat model

- Streambed sediment
- Current fish pop size
- Future Pop size

Combine 2 different models
Representing time in a BBN

Time is difficult to represent in an BBN because they cannot contain feedback loops.

There are, however, 2 basic way to represent time:

To examine the effects of a single decision at various points in time:

Represent time as a node, usually a constant or decision node.

To examine the effects of sequential decisions at various points in time:

Design influence diagram as a sequence of decisions.
Representing time with a constant node

BBN of riparian corridor condition at 10 and 100 years from cessation of cattle grazing
(from Reiman et al. 2001)

At 10 years

Prior Riparian Condition
- Intact: 33.5
- Mod Degraded: 40.8
- Hi Degraded: 25.7

Riparian condition
- Intact: 33.5
- Mod Degraded: 40.8
- Hi Degraded: 25.7

Mitigation
- Hi Mitigation: 33.3
- Mod Mitigation: 33.3
- Low Mitigation: 33.3

At 100 years

Prior Riparian Condition
- Intact low damag: 50.0
- Mod High: 50.0

Riparian condition
- Intact: 42.2
- Mod Degraded: 37.2
- Hi Degraded: 20.7

Mitigation
- Hi Mitigation: 33.3
- Mod Mitigation: 33.3
- Low Mitigation: 33.3

Time = t+10

Time = t+100
Modeling sequential processes

Repeating the same process through time
Mallard population at time $t$ modeled as a function of the previous population size ($t-1$) and the number of ponds at time $t$.

Identical conditional probability table
Parameterizing Bayesian Belief Networks

Model relationships directly from data

Woodpecker model
No data (uniform probabilities)

Woodpecker data file

<table>
<thead>
<tr>
<th>IDnum</th>
<th>Hardwood</th>
<th>Snag</th>
<th>Woodpecker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.62917</td>
<td>0.776599</td>
<td>0.387299</td>
</tr>
<tr>
<td>2</td>
<td>6.035</td>
<td>4.05669</td>
<td>0.947585</td>
</tr>
<tr>
<td>3</td>
<td>8.8726</td>
<td>6.56013</td>
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<td>4</td>
<td>18.7636</td>
<td>2.42272</td>
<td>1.40156</td>
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<tr>
<td>5</td>
<td>6.42409</td>
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<td>6</td>
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<td>5.07255</td>
<td>0.762326</td>
</tr>
</tbody>
</table>
Parameterizing Bayesian Belief Networks

Using existing models

Network representation of linear regression \( Y = 3 + X1 + 0.5X2 + e \)

For \( X1 = 5 \) and \( X2 = 3 \), predicted \( Y = 9.75 \)

Recall linear regression assumptions:

Confidence limits
and when information is completely lacking......

**“Expert” Judgment**

<table>
<thead>
<tr>
<th>Refounding and support</th>
<th>Habitat stability</th>
<th>Insect Diversity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Low</td>
<td>Low, Moderate, High</td>
</tr>
<tr>
<td>Low</td>
<td>Medium</td>
<td>.</td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>.</td>
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</tbody>
</table>

Combine subjective probabilities across experts
On to the case study....