

This note reviews a few differentiation formulae and then provides a number of sample problems (with answers).

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Review ...

In the formulae below  $f$  and  $g$  are differentiable functions and  $a$ ,  $b$  and  $n$  are constants. Note  $\log(x)$ ,  $\ln(x)$  and  $\log_e(x)$  all mean the natural logarithm of  $x$ . Here  $e$  is Euler's number,  $e = 2.718281828459045235 \dots$ . For the logarithm base 10 we will use  $\log_{10}(x)$ .

$$\begin{aligned} \frac{d}{dx}(af(x) + bg(x)) &= af'(x) + bg'(x) \\ \frac{d}{dx}(f(x)g(x)) &= f'(x)g(x) + f(x)g'(x) \\ \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\ \frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\ \frac{d}{dx}(x^n) &= nx^{n-1} \\ \frac{d}{dx}a^x &= a^x \log(a) \quad (a > 0, a \neq 1) \\ \frac{d}{dx}e^x &= e^x \\ \frac{d}{dx}\log_b(x) &= \frac{1}{x \log(b)} = \frac{\log_b(e)}{x} \quad (b > 0, b \neq 1) \\ \frac{d}{dx}\log(x) &= \frac{1}{x} \\ \frac{d}{dx}\sin(x) &= \cos(x) \\ \frac{d}{dx}\cos(x) &= -\sin(x) \\ \frac{d}{dx}\tan(x) &= \sec^2(x) = 1 + \tan^2(x) \end{aligned}$$

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The problems ...

**Problem 1.**  $\frac{d}{dx} \log(1 + e^x)$

**Problem 2.**  $\frac{d}{dx} e^{x^2+2x}$

**Problem 3.**  $\frac{d}{dx} x^x$

- Problem 4.**  $\frac{d}{dx} e^x \sin(x)$
- Problem 5.**  $\frac{d}{dx} \sin(1/x)$
- Problem 6.**  $\frac{d}{dx} \frac{2x^2-3x+4}{x^2+2x+3}$
- Problem 7.**  $\frac{d}{dx} (x^2 + 3x + 4)^{50}$
- Problem 8.**  $\frac{d}{dx} x \sin(\log(x))$
- Problem 9.**  $\frac{d}{dx} (2x + 3)^{10} (x - 2)^8$
- Problem 10.** If  $g(x) = \frac{x^2+x-1}{2x+3}$  and  $f(x) = \frac{x^2+3x+4}{3x^2-2x+1}$  then compute  $(f \circ g)(x)$ . Next compute the derivative directly and also by using the chain rule,
- Problem 11.**  $\lim_{t \rightarrow 0} \frac{\tan(3t)}{t}$  Hint: Express limit as a derivative.
- Problem 12.**  $\frac{d}{dx} \log(x^2 + 1)$
- Problem 13.**  $\frac{d}{dx} 2e^{x^2}$
- Problem 14.**  $\frac{d}{dx} (x \arctan(x) - \frac{1}{2} \log(x^2 + 1))$
- Problem 15.**  $\frac{d}{dx} \arcsin(x^2)$
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The answers ...

- Answer 1.**  $\frac{e^x}{1+e^x}$
- Answer 2.**  $2(x+1)e^{x^2+2x}$
- Answer 3.**  $x^x (\log(x) + 1)$
- Answer 4.**  $e^x (\cos(x) + \sin(x))$
- Answer 5.**  $-\frac{\cos(1/x)}{x^2}$
- Answer 6.**  $\frac{7x^2+4x-17}{(x^2+2x+3)^2}$
- Answer 7.**  $50(2x+3)(x^2+3x+4)^{49}$
- Answer 8.**  $\cos(\log(x)) + \sin(\log(x))$
- Answer 9.**  $4(9x-4)(2x+3)^9(x-2)^7$
- Answer 10.**  $\frac{x^4+8x^3+30x^2+49x+28}{3x^4+2x^3-9x^2+4x+18}$ . The derivative is  $-11 \frac{2x^6+18x^5+51x^4+36x^3-75x^2-144x-70}{(3x^4+2x^3-9x^2+4x+18)^2}$ .
- Answer 11.** 3
- Answer 12.**  $\frac{2x}{x^2+1}$
- Answer 13.**  $2x \log(2) e^{x^2} 2e^{x^2}$
- Answer 14.**  $\arctan(x)$
- Answer 15.**  $\frac{2x}{\sqrt{1-x^4}}$