Partial Fractions

Mth 252 Spring 1998

Bent Petersen

This Maple V5 worksheet illustrates that Maple's convert command, with the options *parfrac* and *fullparfrac*, provides a convenient way to expand rational functions in partial fractions. I simply provide a few examples here - nothing profound.

Example 1

```
> f15:=(x^3+x^2+1)/(x^4+x^3+2*x^2);
```

$$f15 := \frac{x^3+x^2+1}{x^4+x^3+2\,x^2}$$

```
> convert(f15,parfrac,x);
```

$$\frac{1}{2}\,\frac{1}{x^2}-\frac{1}{4}\,\frac{1}{x}+\frac{1}{4}\,\frac{3+5\,x}{x^2+x+2}$$

Example 2

```
> f3:=(x^2+3*x-4)/((2*x-1)^2*(3*x+3));
```

$$f3 := \frac{x^2+3x-4}{(2x-1)^2(3x+3)}$$

```
> convert(f3,parfrac,x);
```

$$-\frac{1}{2}\frac{1}{(2x-1)^2}+\frac{11}{18}\frac{1}{2x-1}-\frac{2}{9}\frac{1}{x+1}$$

Example 3

```
> g1:=(x^3-2*x^2+3*x+5)/((x^2+1)*(x^2-1));
```

$$g1 := \frac{x^3-2x^2+3x+5}{(x^2+1)(x^2-1)}$$

```
> convert(g1,parfrac,x);
```

$$\frac{7}{4}\frac{1}{x-1}+\frac{1}{4}\frac{1}{x+1}-\frac{1}{2}\frac{7+2x}{x^2+1}$$

Example 4

```
> f50:=(x^3-2*x^2+x+1)/(x^4+5*x^2+4);
```

$$f50 := \frac{x^3-2x^2+x+1}{x^4+5x^2+4}$$

```
> convert(f50,parfrac,x);
```

$$\frac{x-3}{x^2+4}+\frac{1}{x^2+1}$$

Example 5

```
> f67:=(4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70);
```

$$f67 := \frac{4\,x^3 - 27\,x^2 + 5\,x - 32}{30\,x^5 - 13\,x^4 + 50\,x^3 - 286\,x^2 - 299\,x - 70}$$

```
> convert(f67,parfrac,x);
```

$$\frac{24110}{4879}\,\frac{1}{5\,x+2} - \frac{668}{323}\,\frac{1}{2\,x+1} - \frac{9438}{80155}\,\frac{1}{3\,x-7} + \frac{1}{260015}\,\frac{48935 + 22098\,x}{x^2 + x + 5}$$

Example 6

```
> f68:=(12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4);
```

$$f68 := \frac{12\,x^5 - 7\,x^3 - 13\,x^2 + 8}{100\,x^6 - 80\,x^5 + 116\,x^4 - 80\,x^3 + 41\,x^2 - 20\,x + 4}$$

```
> convert(f68,parfrac,x);
```

$$\frac{5828}{1815}\,\frac{1}{(5\,x-2)^2} - \frac{59096}{19965}\,\frac{1}{5\,x-2} + \frac{2}{3993}\,\frac{816 + 2843\,x}{2\,x^2+1} + \frac{1}{363}\,\frac{-251 + 313\,x}{(2\,x^2+1)^2}$$

Example 7

Maple trips over some cases:

```
> f78:=(x^4+1)/(x^8+1);
```

$$f78 := \frac{x^4+1}{x^8+1}$$

```
> convert(f78,parfrac,x);
```

$$\frac{x^4+1}{x^8+1}$$

With some help Maple may be able to do expansions that fail without help:

```
> alpha:=I^(1/4);
```

$$\alpha := (-1)^{(1/8)}$$

```
> convert(f78,parfrac,x,alpha);
```

$$\frac{\left(-\frac{1}{8}+\frac{1}{8}I\right)(-1)^{(1/8)}}{x+(-1)^{(5/8)}}+\frac{\left(\frac{1}{8}-\frac{1}{8}I\right)(-1)^{(1/8)}}{x-(-1)^{(5/8)}}+\frac{1}{8}\,\frac{(-1)^{(1/8)}\sqrt{2}}{x+(-1)^{(3/8)}}$$
$$+\frac{1}{8}\,\frac{(-1)^{(5/8)}\sqrt{2}}{x+(-1)^{(7/8)}}-\frac{1}{8}\,\frac{(-1)^{(1/8)}\sqrt{2}}{x-(-1)^{(3/8)}}-\frac{1}{8}\,\frac{(-1)^{(5/8)}\sqrt{2}}{x-(-1)^{(7/8)}}$$
$$+\frac{\left(\frac{1}{8}+\frac{1}{8}I\right)(-1)^{(1/8)}}{x+(-1)^{(1/8)}}+\frac{\left(-\frac{1}{8}-\frac{1}{8}I\right)(-1)^{(1/8)}}{x-(-1)^{(1/8)}}$$

Understanding what is going on above requires some abstract algebra (field theory). Check Maple's help to get started.

Another way to handle difficult cases is to provide the option "complex." In this case maple will perform an approximate floating point partial fraction expansion. Sometimes that sufices.

```
> convert(f78,parfrac,x,complex);
```

$$\frac{.06764951253+.1633203706\,I}{x+.9238795325+.3826834324\,I}+\frac{.06764951258-.1633203706\,I}{x+.9238795325-.3826834324\,I}+\frac{.1633203706+.06764951255\,I}{x+.3826834324+.9238795325\,I}+\frac{.1633203705-.06764951255\,I}{x+.3826834324-.9238795325\,I}+\frac{-.1633203706+.06764951253\,I}{x-.3826834324+.9238795325\,I}+\frac{-.1633203705-.06764951247\,I}{x-.3826834324-.9238795325\,I}+\frac{-.06764951250+.1633203706\,I}{x-.9238795325+.3826834324\,I}+\frac{-.06764951252-.1633203706\,I}{x-.9238795325-.3826834324\,I}$$

Another possibility is to try the "fullparfrac" conversion. Again understanding what is going on requires some abstract algebra (field theory). Check Maple's help to get started.

```
> convert(f78,fullparfrac,x);
```

$$\sum_{\_\alpha = \%1} \frac{-\frac{1}{8}\_\alpha^5 - \frac{1}{8}\_\alpha}{x - \_\alpha}$$

$$\%1 := \mathrm{RootOf}(\_Z^8 + 1)$$

```
>
```