Mth 256 Fall 2000

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Two illustrations of Newton's law of cooling

In the graphs the red is the ambient temperature (which drives the system) and the blue is the temperature of our wine cellar, root cellar, or whatever (the system response). The numbers are not realistic.

Notice how the response tracks the ambient with a time delay (phase shift in the first example) and with a reduced amplitude of fluctuation.

```
> restart;
> ode:=diff(T(t),t)+k*T(t)=k*F(t);
```

$$ode := \left(\frac{\partial}{\partial t}\mathrm{T}(t)\right) + k\,\mathrm{T}(t) = k\,\mathrm{F}(t)$$

```
> F:=t->B+A*cos(omega*t);
```

$$F := t \to B + A\cos(\omega\, t)$$

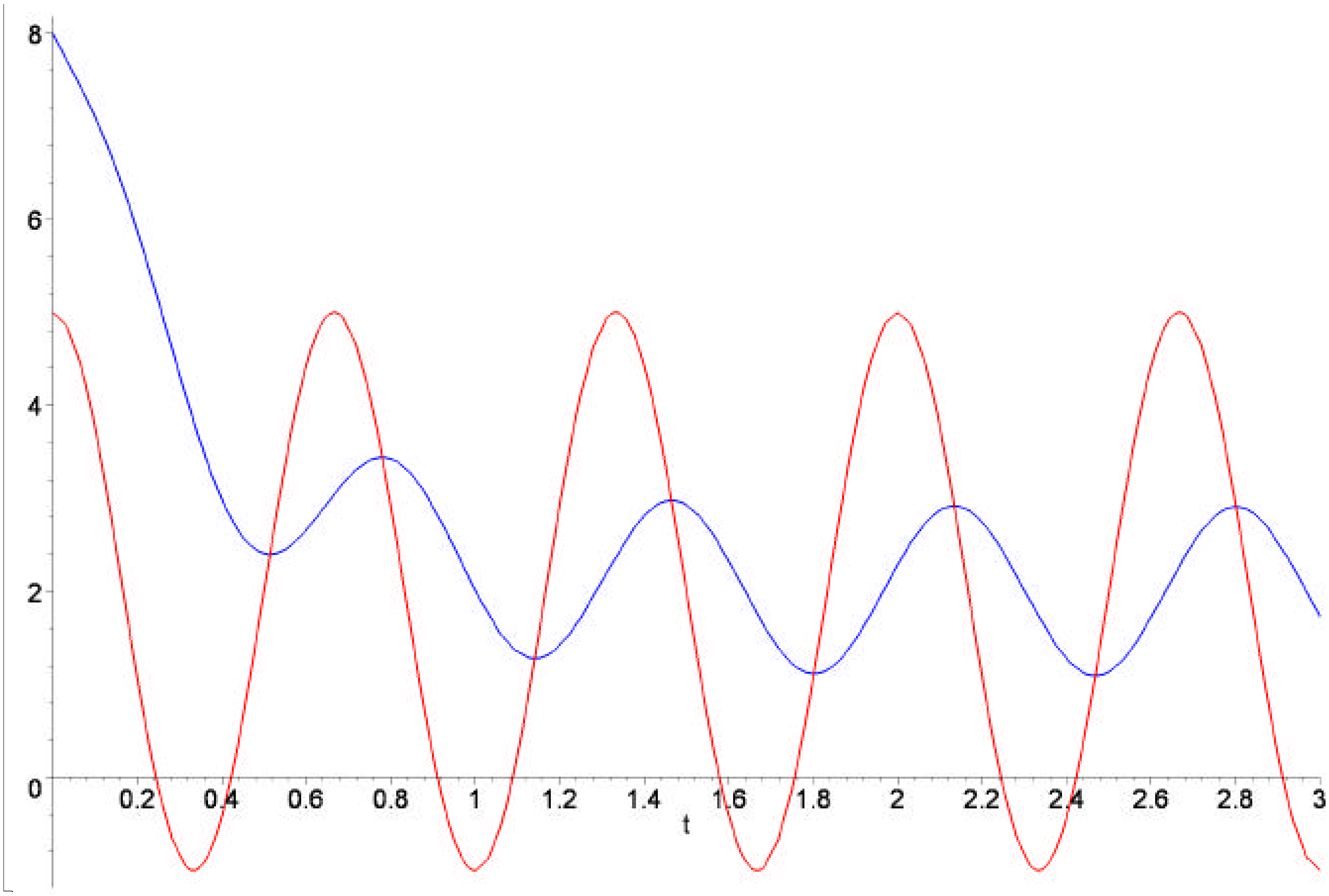
```
> dsolve(ode):Ts:=unapply(rhs(%),t);
```

$$Ts := t \to \frac{B\,k^2 + B\,\omega^2 + k^2 A\cos(\omega\, t) + k\,A\,\omega\sin(\omega\, t) + \mathbf{e}^{(-k\,t)}\,\_C1\,k^2 + \mathbf{e}^{(-k\,t)}\,\_C1\,\omega^2}{k^2 + \omega^2}$$

```
> B:=2:A:=3:k:=3:omega:=3*Pi:
> solve(Ts(0)=8,_C1): _C1:=%;
```

$$\_C1 := 3\,\frac{1 + 2\,\pi^2}{1 + \pi^2}$$

```
> plot([F(t),Ts(t)],t=0..3,color=[red,blue],thickness=3);
```




```
> restart;
> ode:=diff(T(t),t)+k*T(t)=k*F(t);
```

$$ode := \left(\frac{\partial}{\partial t}\mathrm{T}(t)\right) + k\,\mathrm{T}(t) = k\,\mathrm{F}(t)$$

```
> F:=t->B+A*(Heaviside(t-1)-Heaviside(t-2))-A*(Heaviside(t-2)-Heaviside(t-3));
```

$$F := t \rightarrow B + A\,(\mathrm{Heaviside}(t-1) - \mathrm{Heaviside}(t-2)) - A\,(\mathrm{Heaviside}(t-2) - \mathrm{Heaviside}(t-3))$$

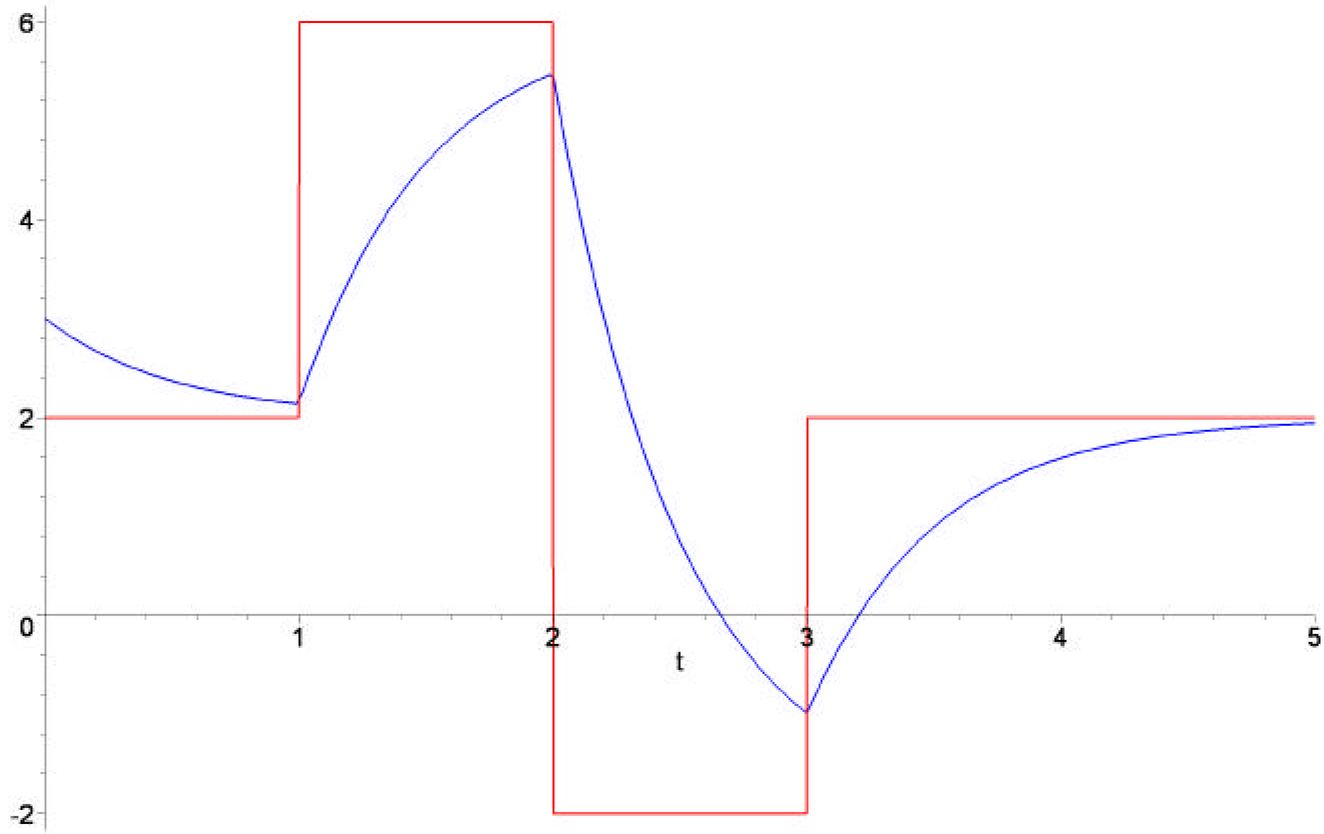
```
> dsolve(ode):Ts:=unapply(rhs(%),t);
```

$$Ts := t \rightarrow B + A\,\mathrm{Heaviside}(t-1) - A\,\mathrm{Heaviside}(t-1)\,\mathbf{e}^{(-k\,t+k)} - 2\,A\,\mathrm{Heaviside}(t-2) + 2\,A\,\mathrm{Heaviside}(t-2)\,\mathbf{e}^{(-k\,t+2\,k)} + A\,\mathrm{Heaviside}(t-3) - A\,\mathrm{Heaviside}(t-3)\,\mathbf{e}^{(-k\,t+3\,k)} + \mathbf{e}^{(-k\,t)}\,\_C1$$

```
> B:=2:A:=4:k:=2:omega:=3*Pi:
> solve(Ts(0)=3,_C1): _C1:=%;
```

$$\_C1 := 1$$

```
> plot([F(t),Ts(t)],t=0..5,color=[red,blue],thickness=3);
```


6
4
2
0
-2
1
2
3
4
5
t

>