Complex Numbers - Early History
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The introduction of complex numbers to a large extent was motivated by the problem of finding roots of polynomials.

Scipione dal Ferro solved some cubic equations around 1500 and confided his method to Antonio Maria Fior around 1510. In 1535 Fior, who must have been playing with dal Ferro's method, challenged Tartaglia (Niccoló Fontana) to solve 30 cubics. Tartaglia did so. It is sobering to realize that this work was done before the introduction of symbolic coefficients and abstract algebra.

In 1539 Gerolamo Cardano pressed Tartaglia to reveal his method and he promised to keep it secret. However Cardano determined that Tartaglia's method was the same as dal Ferro's and in 1542 he published it in spite of his promise to keep it secret. He had after all, not made any promise to dal Ferro!

Cardano's publication includes a detailed proof of the method in numerous special cases. The modern version of the Ferro-Tartaglia-Cardano method is usually called Cardano's method. The quartic (degree 4) equation was solved by Cardano's student, Lodovico Ferrari, and was also published by Cardano.

Using Cardano's method for a real cubic with 3 real roots we must, as part of the calculation, deal with a quadratic with complex roots and take cubic roots of these complex numbers! The complex numbers drop out in the final answer, but they do, and must occur! Cardano actually observed this fact (calling the cubic "irreducible" in this case), but he missed the opportunity to follow up and to make a serious study of complex numbers. It was just too soon!

Curiously Cardano also regarded negative solutions as "fictitious" but gave the negative solutions in every case, though he missed the opportunity to put forward negative solutions as real. In fact he avoided negative numbers as much as possible. Thus he would write the equation $x^3 - 28x + 48 = 0$ as $x^3 + 48 = 28x$ and therefore have a multiplicity of unnecessary special cases.

Around 1590 François Vieta discovered a trigonometric approach to finding roots of cubics. This method eliminates the intermediate complex numbers. That was probably unfortunate for the development of a theory of complex numbers! Vieta also did something much more important - he introduced the idea of symbolic coefficients for polynomials.

In Vieta's work the symbolic coefficients were still regarded as positive numbers, but who is going to check every newly calculated coefficient? Symbolic calculations hid the negative numbers and therefore made them gradually more acceptable, though Vieta discarded negative solutions entirely. The restriction to positive numbers was due to viewing numbers as geometric quantities (though even in geometry the notion of orientation can lead to negative numbers).

The real birth of complex numbers and the recognition that they are every bit as “concrete” as positive real numbers took place in early 1800’s with the introduction of the complex plane in the work of Caspar Wessel (c. 1797), Jean Robert Argand (c. 1806), John Warren (c. 1828), and Carl Friedrich Gauss (c. 1831).

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