Sample Laplace transforms and Laplace exchange formulæ

for Mth 256, Applied Differential Equations[1]

Some Laplace transforms

$$\begin{aligned}
\mathcal{L}\left\{1\right\}(s) &= \frac{1}{s} \\
\mathcal{L}\left\{\mathrm{e}^{at}\right\}(s) &= \frac{1}{s-a} \\
\mathcal{L}\left\{t^{n}\right\}(s) &= \frac{n!}{s^{n+1}} \\
\mathcal{L}\left\{\cos\omega t\right\}(s) &= \frac{s}{s^2+\omega^2} \\
\mathcal{L}\left\{\sin\omega t\right\}(s) &= \frac{\omega}{s^2+\omega^2} \\
\mathcal{L}\left\{\mathrm{e}^{at}\cos\omega t\right\}(s) &= \frac{s-a}{(s-a)^2+\omega^2} \\
\mathcal{L}\left\{\mathrm{e}^{at}\sin\omega t\right\}(s) &= \frac{\omega}{(s-a)^2+\omega^2} \\
\mathcal{L}\left\{\mathrm{e}^{at}\cosh\nu t\right\}(s) &= \frac{s-a}{(s-a)^2-\nu^2} \\
\mathcal{L}\left\{\mathrm{e}^{at}\sinh\nu t\right\}(s) &= \frac{\nu}{(s-a)^2-\nu^2} \\
\mathcal{L}\left\{\sqrt{t}\right\}(s) &= \frac{\sqrt{\pi}}{2s^{3/2}} \\
\mathcal{L}\left\{t^{n}\mathrm{e}^{at}\right\}(s) &= \frac{n!}{(s-a)^{n+1}} \\
\mathcal{L}\left\{u(t-a)\right\}(s) &= \frac{\mathrm{e}^{-as}}{s} \quad (u = \text{ unit step}) \\
\mathcal{L}\left\{\delta(t-a)\right\}(s) &= \mathrm{e}^{-as} \quad (\delta = \text{ Dirac delta})
\end{aligned}$$

If f is periodic with period $T > 0$ then $\mathcal{L}\left\{f(t)\right\} = \dfrac{\int_0^T \mathrm{e}^{-st}\, f(t)\, \mathrm{d}t}{1-\mathrm{e}^{-sT}}$.

$$\begin{aligned}
\mathcal{L}\left\{1+\sum_{k=1}^{\infty}(-1)^k\, u(t-k)\right\}(s) &= \frac{1}{s\left(1+\mathrm{e}^{-s}\right)} \\
\mathcal{L}\left\{\left|\sin(t)\right|\right\}(s) &= \frac{\coth\left(\frac{\pi s}{2}\right)}{1+s^2}
\end{aligned}$$

[1] B. Petersen, Mth 256 Fall 2000

Some Laplace exchange formulæ

If $\mathcal{L}\{f(t)\}(s) = F(s)$ then

$$\begin{aligned}
\mathcal{L}\left\{e^{at} f(t)\right\}(s) &= F(s-a) \\
\mathcal{L}\{t^n f(t)\}(s) &= (-1)^n F^{(n)}(s) \\
\mathcal{L}\left\{\frac{f(t)}{t}\right\}(s) &= \int_s^\infty F(r)\ \mathrm{d}r \quad \left(\text{if } \frac{f(t)}{t} \text{ integrable at } 0\right) \\
\mathcal{L}\left\{\int_0^t f(r)\ \mathrm{d}r\right\}(s) &= \frac{F(s)}{s} \\
\mathcal{L}\left\{\frac{df}{dt}\right\}(s) &= sF(s) - f(0) \quad (\text{if } f \text{ cont. on } [0,\infty)) \\
\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\}(s) &= s^2 F(s) - sf(0) - f'(0) \quad \big(\text{if } f,\ f' \text{ cont. on } [0,\infty)\big) \\
\mathcal{L}\{u(t-a)f(t-a)\}(s) &= e^{-as} F(s) \quad (u = \text{ unit step}) \\
\mathcal{L}\{f(at)\}(s) &= \frac{1}{a} F\left(\frac{s}{a}\right).
\end{aligned}$$

If $\mathcal{L}\{f(t)\}(s) = F(s)$ and $\mathcal{L}\{g(t)\}(s) = G(s)$ then $\mathcal{L}\{(f*g)(t)\}(s) = F(s)G(s)$ where $f*g$ is defined by $(f*g)(t) = \int_0^t f(t-r)g(r)\ \mathrm{d}r$.

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