Linear Differential Equations with Heaviside and Dirac-delta term: Examples

Mth 256 November 29 2000

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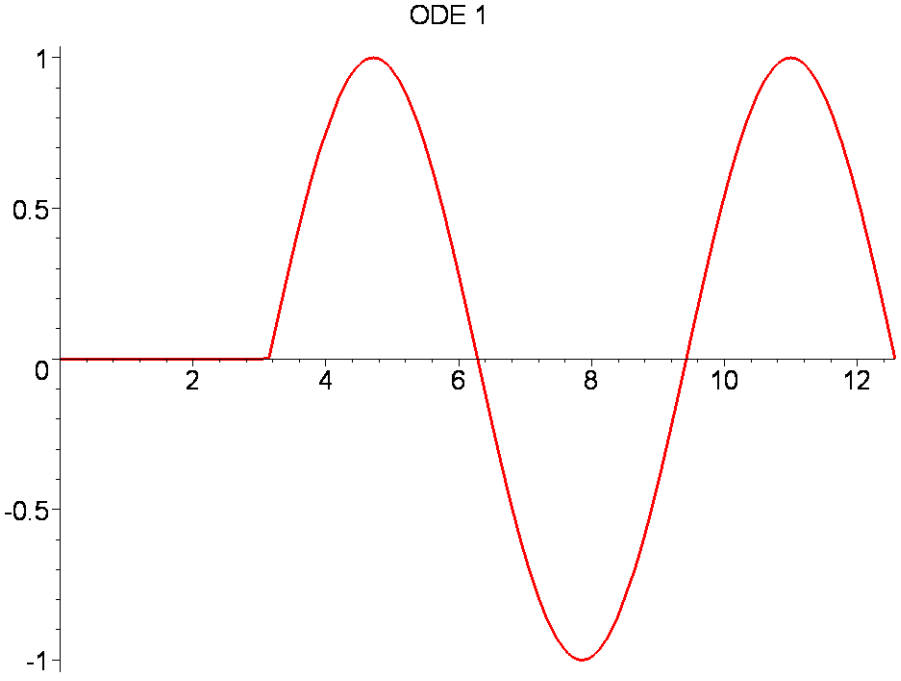
Filename: 256fall2000_sample_ode_dirac.mws

```
> restart;
```

```
> ode1:=diff(y(t),t,t)+y(t)=Dirac(t-Pi);
> inits1:=y(0)=0,D(y)(0)=0:
> dsolve({ode1,inits1},y(t)): soln1:=unapply(rhs(%),t);
> plot(soln1,0..4*Pi,thickness=3,title="ODE 1");
```

$$ode1 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + y(t) = \text{Dirac}(t-\pi)$$

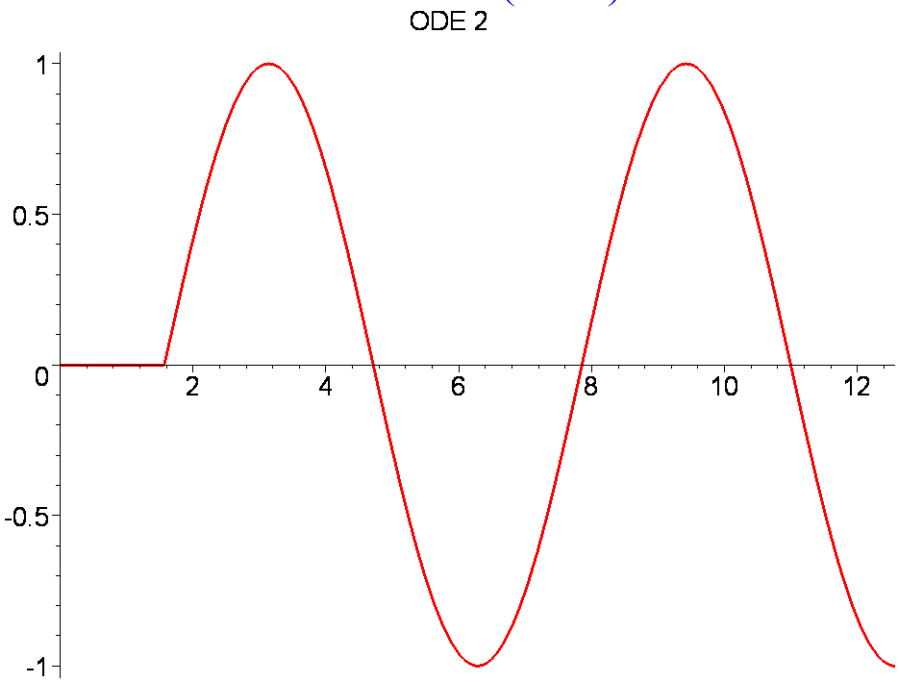
$$soln1 := t \to -\text{Heaviside}(t-\pi)\sin(t)$$




```
> ode2:=diff(y(t),t,t)+y(t)=Dirac(t-Pi/2);
> inits2:=y(0)=0,D(y)(0)=0:
> dsolve({ode2,inits2},y(t)): soln2:=unapply(rhs(%),t);
> plot(soln2,0..4*Pi,thickness=3,title="ODE 2");
```

$$ode2 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + y(t) = \text{Dirac}\left(t-\frac{1}{2}\pi\right)$$

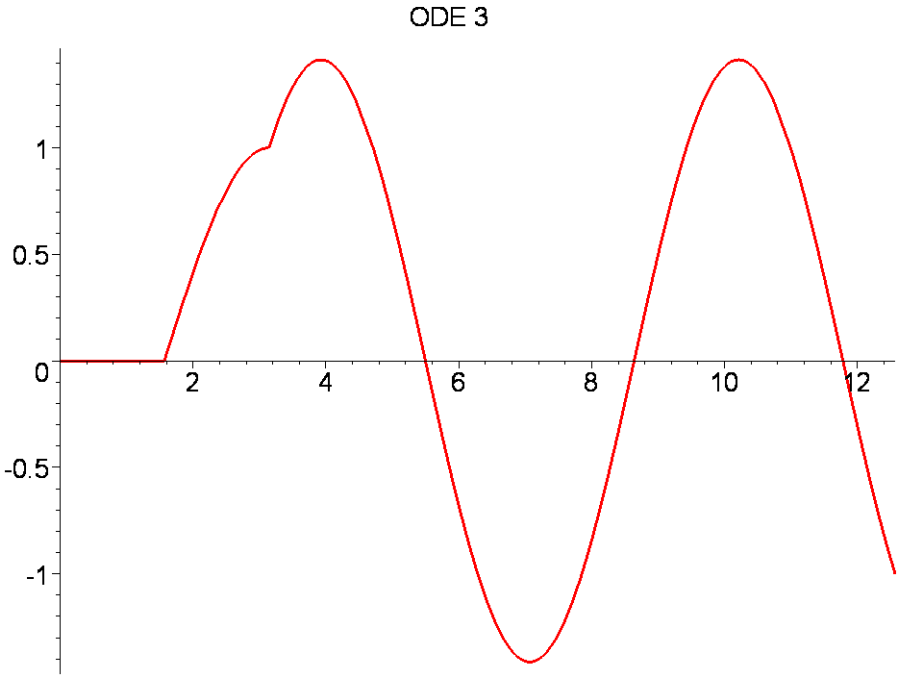
$$soln2 := t \to -\text{Heaviside}\left(t - \frac{1}{2}\pi\right)\cos(t)$$




```
> ode3:=diff(y(t),t,t)+y(t)=Dirac(t-Pi/2)+Dirac(t-Pi);
> inits3:=y(0)=0,D(y)(0)=0:
> dsolve({ode3,inits3},y(t)): soln3:=unapply(rhs(%),t);
> plot(soln3,0..4*Pi,thickness=3,title="ODE 3");
```

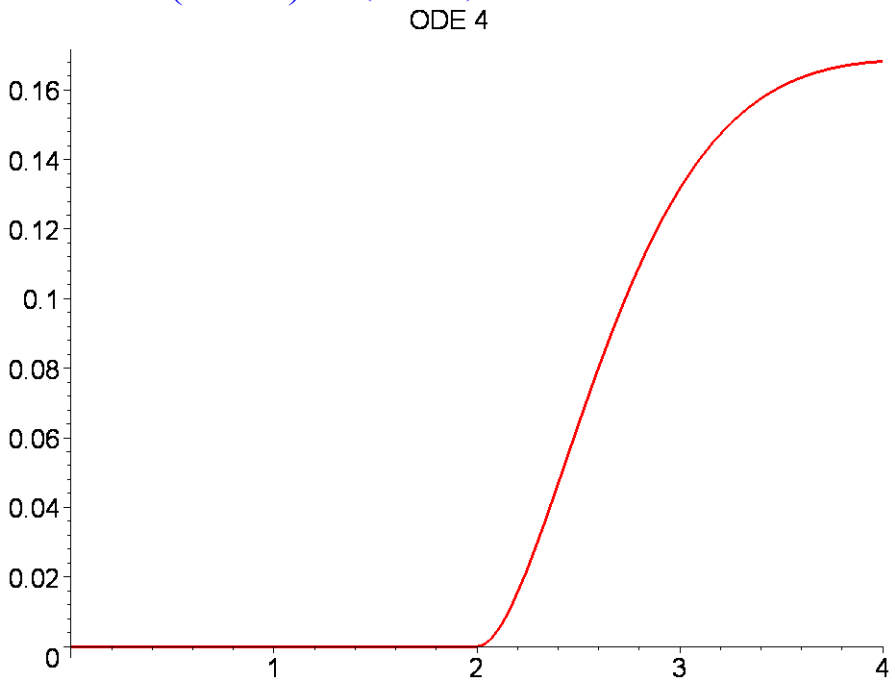
$$ode3 := \left(\frac{\partial^2}{\partial t^2}\,y(t)\right) + y(t) = \text{Dirac}\left(t - \frac{1}{2}\pi\right) + \text{Dirac}(t - \pi)$$

$$soln3 := t \to -\text{Heaviside}\left(t - \frac{1}{2}\pi\right)\cos(t) - \text{Heaviside}(t - \pi)\sin(t)$$




```
> ode4:=diff(y(t),t,t)+4*diff(y(t),t)+6*y(t)=Heaviside(t-2);
> inits4:=y(0)=0,D(y)(0)=0:
> dsolve({ode4,inits4},y(t)): soln4:=unapply(rhs(%),t):
> plot(soln4,0..4,thickness=3,title="ODE 4");
```

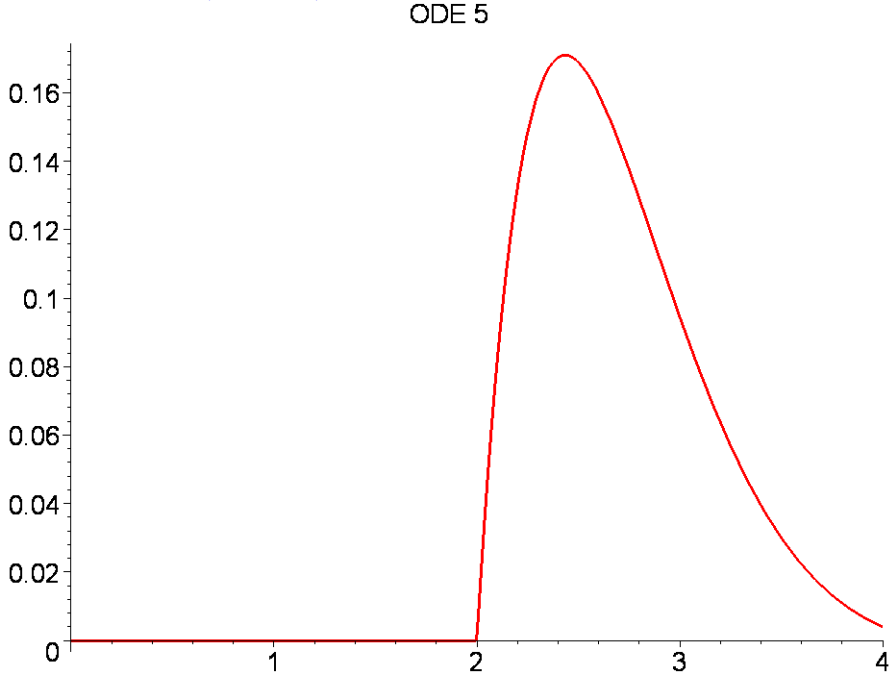
$$ode4 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\left(\frac{\partial}{\partial t} y(t)\right) + 6\, y(t) = \text{Heaviside}(t-2)$$




```
> ode5:=diff(y(t),t,t)+4*diff(y(t),t)+6*y(t)=Dirac(t-2);
> inits5:=y(0)=0,D(y)(0)=0:
```

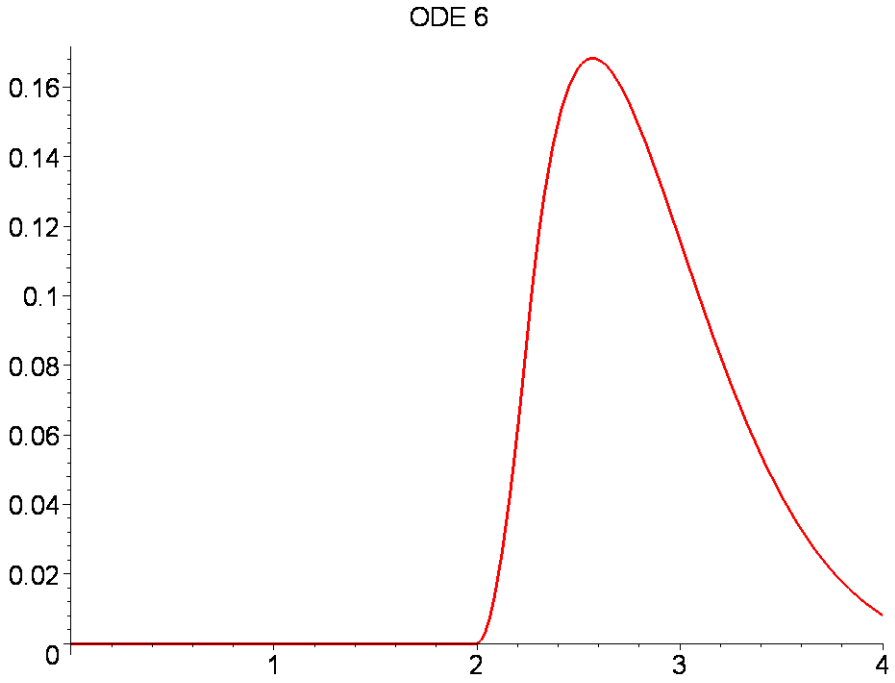
```
> dsolve({ode5,inits4},y(t)): soln5:=unapply(rhs(%),t):
> plot(soln5,0..4,thickness=3,title="ODE 5");
```

$$ode5 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\left(\frac{\partial}{\partial t} y(t)\right) + 6\, y(t) = \text{Dirac}(t-2)$$




```
> ode6:=diff(y(t),t,t)+4*diff(y(t),t)+6*y(t)=4*(Heaviside(t-2)-Heavi
  side(t-9/4));
> inits6:=y(0)=0,D(y)(0)=0:
> dsolve({ode6,inits6},y(t)): soln6:=unapply(rhs(%),t):
> plot(soln6,0..4,thickness=3,title="ODE 6");
```

$$ode6 := \left(\frac{\partial^2}{\partial t^2} y(t)\right) + 4\left(\frac{\partial}{\partial t} y(t)\right) + 6\, y(t) = 4\,\text{Heaviside}(t-2) - 4\,\text{Heaviside}\left(t-\frac{9}{4}\right)$$

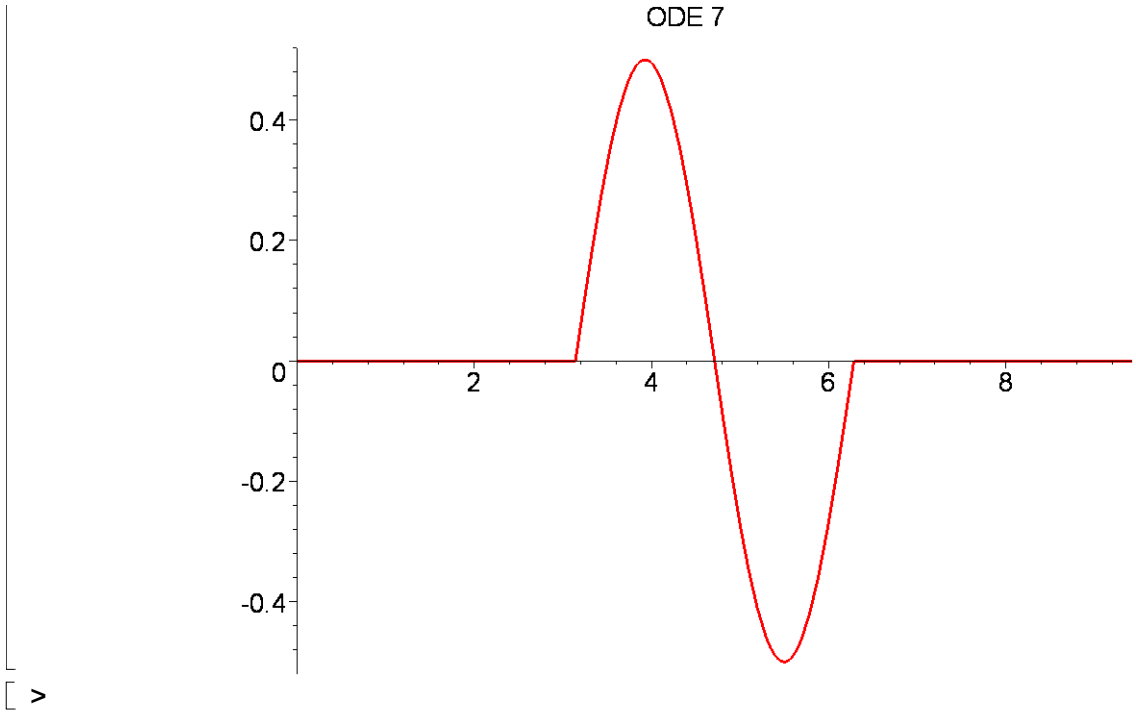



```
> ode7 := diff(y(t),`$`(t,2))+4*y(t) = Dirac(t-Pi)-Dirac(t-2*Pi);
> inits7:=y(0)=0,D(y)(0)=0;
> dsolve({ode7,inits7},y(t)): soln7:=unapply(rhs(%),t);
> plot(soln7,0..3*Pi,thickness=3,title="ODE 7");
```

$$ode7 := \left(\frac{\partial^2}{\partial t^2}\,y(t)\right) + 4\,y(t) = \mathrm{Dirac}(t-\pi) - \mathrm{Dirac}(t-2\,\pi)$$

$$inits7 := y(0) = 0,\ D(y)(0) = 0$$

$$soln7 := t \to -\frac{1}{2}\left(-\mathrm{Heaviside}(t-\pi) + \mathrm{Heaviside}(t-2\,\pi)\right)\sin(2\,t)$$


ODE 7
0.4
0.2
0
-0.2
-0.4
2
4
6
8

>