Applied Differential Equations – Mth 256

Archive – Spring 1992 Files

Jan 12, 2001

This archive contains the tests from Mth 256 Spring 1992. The original test instructions, headers and formatting have not been preserved.

Contents

1 Test 1

Problem 1. Solve the initial value problem

$$\frac{dy}{dx} = y(1+x^2) - (1+x^2), \quad y(0) = 2.$$

Problem 2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{4x^2+y^2}{2xy}, \quad y(2) = 2.$$

Problem 3. For what values of p and q is $x^p y^q$ an intgrating factor for the ordinary differential equation

$$(6y^2 + 3y - 4xy)\,dx + (-3x^2 + 3x + 8xy)\,dy = 0?$$

Problem 4. Solve the *exact* ordinary differential equation

$$(2xy^3 - y^2 - 2)\,dx + (3x^2y^2 - 2xy + 3)\,dy = 0.$$

Problem 5. Find the family of orthogonal trajectories to the one-parameter family of hyperbolas given by $2y^2 - x^2 = \alpha$.

Problem 6. A brine solution consisting of 0.06 oz/gal salt dissolved in water flows into a large tank at the rate 3.0 gal/min. The solution inside the tank is kept well-mixed and flows out of the tank at the rate 2.0 gal/min. If the tank initially contains 50.0 gal of brine of concentration 0.03 oz/gal determine the amount of salt in the tank after t minutes. When will the concentration of salt in the tank reach 0.05 oz/gal? Assume the tank is so large that it does not overflow.

Problem 7. For what value of λ is $y = x^\lambda \log(x)$ a solution of the ordinary differential equation $x^2y'' - 5xy' + 9y = 0$?

2 Test 2

Problem 8. For each of the following second order constant coefficient linear ordinary differential equations of order two find the characteristic roots and the general solution.

(A)	$y'' - 4y' = 0$	(D)	$y'' + 4y = 0$	(G)	$y'' = 0$
(B)	$y'' - 4y = 0$	(E)	$y'' + 4y' + 4y = 0$	(H)	$y'' - y' + y = 0$
(C)	$y'' + 4y' = 0$	(F)	$y'' + 4y' + 13y = 0$		

Problem 9. The differential equation

$$x^2y'' - 2y = 4x^3 \log(x)$$

has the complementary solution $y_c = c_1x^2 + c_2x^{-1}$. Use the method of *variation of parameters* to find a particular solution. Then find the general solution.

Problem 10. Use the method of *undetermined coefficients* to find a particular solution of the differential equation

$$y'' - y = x^3e^{-x}.$$

Problem 11. Use the method of *undetermined coefficients* to find a particular solution of the differential equation

$$y'' - 2y' + 5y = e^x \sin(2x).$$

Problem 12. Solve the initial value problem

$$y'' = yy' \qquad y(0) = 1, \quad y'(0) = 1.$$

3 Final Exam

Problem 13. A 100 gal tank initially contains 20 gal of brine of concentration 0.24 oz/gal salt. Brine of concentration 0.18 oz/gal flows into the tank at 3 gal/min and the well-mixed solution is drawn off at the rate of 1 gal/min. Find the amount of salt in the tank at the very moment that it begins to overflow.

Problem 14. Find the family of orthogonal trajectories to the one-parameter family of cubics

$$y = \alpha x^3, \qquad \alpha = \text{ arbitrary constant.}$$

Problem 15. Find an integrating factor which depends only on y and then solve the differential equation

$$(2y + y^2 - 6xy)\,dx \;+\; (4x + 3xy - 6x^2)\,dy = 0.$$

Problem 16. The differential equation

$$x^2y'' - 2xy' + 2y = 2x^3 - x^2 + x$$

has the *complementary solution* $c_1x + c_2x^2$. Use VARIATION OF PARAMETERS to find a *particular solution*.

Problem 17. Given that $y_1 = e^x$ is a solution of the ordinary differential equation

$$(2x + 3)y'' + (-2x + 1)y' - 4y = 0$$

use REDUCTION OF ORDER to find a second solution y_2 such that $c_1y_1 + c_2y_2$ is a fundamental solution.

Problem 18. Find the general solution

$$\text{(A)}\quad 2y'' - y' + y = 0 \qquad \text{(B)}\quad y'' + 4y' + 13y = 0 \qquad \text{(C)}\quad y'' - 6y' + 9y = 0.$$

Problem 19. Use the METHOD OF UNDETERMINED COEFFICIENTS to find a particular solution

$$y'' + 2y' - 3y = (x + 1)\,e^x.$$

Problem 20. Find the LAPLACE transform of the solution to the initial value problem

$$y'' + 4y' = u(t - \pi), \quad y(0) = 1, \quad y'(0) = -1$$

where u is the unit step function.

Problem 21. Find the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{2s + 1}{s^2 - 2s - 3}\right\}.$$

Problem 22. Find the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{2s^2 - s + 2}{(s + 2)(s^2 + 2s + 5)}\right\}.$$

4 Make–Up Exam

Four problems on the make-up exam were identical to problems on the exam. Those four problems have been omitted in this section.

Problem 23. A 100 gal tank initially contains 40 gal of brine of concentration 0.24 oz/gal salt. Brine of concentration 0.18 oz/gal flows into the tank at 3 gal/min and the well-mixed solution is drawn off at the rate of 1 gal/min. Find the amount of salt in the tank at the very moment that it begins to overflow.

Problem 24. Find the family of orthogonal trajectories to the one-parameter family of parabolas

$$y = \alpha x^2, \qquad \alpha = \text{ arbitrary constant.}$$

Problem 25. Given that $y_1 = e^x$ is a solution of the ordinary differential equation

$$xy'' - (3x+1)y' + (2x+1)y = 0$$

use REDUCTION OF ORDER to find a second solution y_2 such that $c_1y_1 + c_2y_2$ is a fundamental solution.

Problem 26. Use the METHOD OF UNDETERMINED COEFFICIENTS to find a particular solution

$$y'' + 2y' - 3y = (x^2 - 3)\, e^x.$$

Problem 27. Find the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{3s-7}{s^2-2s-3}\right\}.$$

Problem 28. Find the inverse LAPLACE transform

$$\mathcal{L}^{-1}\left\{\frac{2s^2+1}{(s+2)(s^2+2s+5)}\right\}.$$

5 Contact Information

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